In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institute shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the Dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

7/25/68

PERISHABLE, SEASONAL INVENTORY CONTROL: AN INDUSTRIAL DYNAMICS ANALYSIS

A THESIS

Presented to

The Faculty of the Division of Graduate

Studies and Research

By

James Lawrence Adams

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in the

School of Industrial and Systems Engineering

Georgia Institute of Technology

December, 1971

PERISHABLE, SEASONAL INVENTORY CONTROL:

AN INDUSTRIAL DYNAMICS ANALYSIS

Approved:

Chairman

Date approved by Chairman: 9 Dec 71

ACKNOWLEDGEMENTS

Appreciation is gratefully acknowledged to those who helped me in the preparation of this research. In particular I am grateful to Dr. Joseph J. Talavage and Professor Willard R. Fey, members of the reading committee.

Special thanks go to Professor Fey for developing my interest in industrial dynamics. This thesis could not have been done without his excellent advice, encouragement, and patience.

Sincere appreciation is given to my advisor, Dr. Leslie G. Callahan, Jr., for his capable direction and careful review of this thesis.

Last, I am particularly indebted to my wife Marylee for her patience, understanding, and tolerance during my graduate work. To her I express my special thanks.

TABLE OF CONTENTS

TABLE OF CONTENTS (Concluded)

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Continued)

SUMMARY

The problems of managerial control of inventory systems take on unique time dependent characteristics when dealing with perishable, seasonal goods. The manager must define his controlling policies to be continuously adaptive, minimizing perishability loss while minimizing lost sales.

This research effort was undertaken to better understand the internal causal relationships of such a system by expanding the classical retail store model presented by Jay Forrester (16) to include perishable, seasonal goods. Initially the significant feedback loops were identified and a model of the system was developed. Output from a simulation of the model was compared with hypothetical system behavior patterns, and the model's sensitivity to changing selected variables and parameters was analyzed.

The results of this study provide a reasonably valid representation of the interdependent components of a seasonal, perishable inventory control system. When coupled with a companion cash flow model and further validation by implementation, this model will provide the manager with a better understanding of the most significant aspects of his system, enabling him to better control its output.

CHAPTER I

INTRODUCTION

The general purpose of this research is to develop a means of conceptualizing the internal causal relationships of a certain type inventory system and to provide that system's manager a tool for improving his policy decision processes. The particular system under consideration is a perishable, seasonal inventory control system.

In addition to the problems normally associated with inventory control, systems which include perishable, seasonal products are compounded by the increased significance of time dependence. Although systems dealing with more durable products are also time dependent, the nature of this dependence is usually manifest in terms of fixed periods such as lead time and production lag time. Little emphasis is placed on inventory holding time, and that problem, if considered significant, is most always dealt with by a general First In First Out (FIFO) policy. Of greater importance has been the problem of determining such variables as the economic order quantity (EOQ), reorder point, and buffer stock.

Perishable, seasonal inventory systems are also confronted with the classical inventory control problems. More meaningful, however, is the production rate, or the rate at which the product is available to the consumer. The reorder point varies with the season, and the EOQ is associated more with satisfying such constraints as perishability rate and demand rate than with batch replacement of depleted stock.

Stock-outs are also to be avoided to minimize lost sales. But the constraints involved with determining buffer stock size now include the cost of perishability loss as well as traditional holding costs such as storage space, capital investment, and obsolescence.

The problem of perishability is not new, and considerable effort has been applied toward analyzing means of dealing with this characteristic. Studies involving various storage temperatures, packaging materials, humidity, handling and so forth have been conducted (39, 40, 41, 42, 44, 45). Generally the results of these studies indicate a preferred means of temporarily retarding the inevitable. Perishability retardation is certainly of interest to the system manager, and the time-cost trade off for storing perishable inventory takes on considerable dimension. The size of this dimension depends on its relative significance within the system, and the need for some means of measuring this and the significance of other interdependent relationships within the system becomes apparent.

The characteristics of perishability are manifest in still other aspects of the system. The problem of backorders raises some interesting questions with which the manager must deal. Backordering may be a designed managerial policy wherein orders are accepted but not immediately filled. This is most appropriate in the case of seasonal goods. Backordering may also result from stock-outs where the consumer is satisfied to await delivery of goods not immediately available. In the former case the programmed backordering is best designed if the manager has some control over production rate. Control may be exercised either directly, where the production facility is part of the overall system, or indirectly where production and inventory are separate entities such as farm and market. In either case the manager's control is limited by the capacity of the production facility, and he may plan to satisfy backorders beyond that limit only by assuming additional cost. The characteristic which makes this significantly different from a durable product backorder policy is that the production starting date is constrained lest the product perish before its due date to the consumer. Now the capacity constraint looms even more significant. Furthermore, when backordering is to avoid lost sales in the case of stock-outs, some portion of production capacity may be necessarily allocated to meet this eventuality. And an increasingly greater portion of production capacity must be allocated to provide on-hand inventory as the product comes in season. Before deciding upon any firm operating policy the system manager must have some means of conceptualizing the varying degree of significance of each of these situations in the time domain.

The purpose of this research then is to provide the manager of a perishable, seasonal inventory control system with a means of observing this dynamic situation as it exists and testing the hypothesis that his policies are indeed optimum given his unique situation.

CHAPTER II

LITERATURE SURVEY

The problem of establishing the optimum inventory system is as dynamic as the types of inventory systems are varied. The literature abounds with various schemes for optimizing some one or several functions in any number of given inventory systems. No attempt will be made here at a detailed survey of the plethora of articles and texts that have been published in the past two decades on this subject. Rather, the following is a pursuance of the methodological evolution of inventory optimization procedures and its relationship to the perishable, seasonal inventory system under analysis.

Classical

One of the earliest contributions to the current inventory modeling methodology was presented by Arrow, Harris, and Marschak (4). They determined the best maximum stock and reorder point as functions of demand, the cost of making an order, and the penalty of stock out. In this paper the storage cost is related to the size of inventory via constant parameters, while rate of demand is a random variable. Wagner and Whitin (36) extended the methodology to include varying demands, inventory holding charges, and set up costs for a single item over n periods. This dynamic version of the EOQ model (36) asserts that one does not both place orders and carry inventory forward in the same period. Holding costs are allowed to vary with the size of inventory carried but no provision is made to reduce holding costs resulting from perished inventory reduction and subsequent additional lost inventory costs.

An interesting and innovative contribution to the methodology has recently been provided by Zangwill (37). Following the evolutionary progress of optimization through mathematical programming he devised a network approach for solving the EOQ model. This approach provides a means of revealing the underlying structure of the model facilitating the development of efficient dynamic programming algorithms for determining optimality. This paper is particularly significant in that it represents the value of combining the technique of mathematical programming with the technique of reticulation long used principally by systems engineers. Notably, using the network approach, Zangwill accomplishes an objective similar to that of this research - provide a means of conceptualizing an otherwise extremely complex model which facilitates determining optimality.

In two papers recently presented by Bulinshaya (8,9) the technique of dynamic programming is brought to bear directly on the problem of inventory policies for perishable goods. In his earlier work he assumes linear cost functions for replenishment cost, insufficiency of goods cost (stock out), and storage cost. In contrast his later paper allows replenishment cost to be a convex function of the amount of the order. In each paper he assumes that goods remaining unused at the end of each period n perish. These assumptions rather limit the application of the proposed model. Perishability as noted in a recent paper by Belson and Fleischer (6), is actually modeled by a function similar to that depicted in Figure 1 , rather than the step function proposed by Bulinshaya. According to Belson and Fleischer, during the period of time the product is actually marketable (A) its utility loss is nearly approximated by a convex function. It is observed here the mathematical similarity to an exponential decay.

System's Approach

Coincident with the refinement of mathematical programming application to the classical inventory control models, the theory of cybernetics has evolved to embrace inventory control through servomechanism theory. In a paper by Simon (31) servomechanism theory was applied as a tool for controlling production rate of a single product. A control analogous to a thermostat was devised to automatically control production rate thus providing sufficient inventory to satisfy demands on the system. Although the inventory control system was now defined in the time domain no attention was given to providing for continuous inventory drain due to perishability loss. The system output, or actual inventory, was defined as a function of production rate and customer's order-rate. Perishability rate was not included.

More recently Bather (5) suggested the use of a continuous time analogue form of the inventory control model. In this model

> . . . we suppose that the holding and shortage costs associated with the inventory are determined by a continuous non-negative function $\ell(x)$, where $\ell(x)$ represents the instantaneous cost per unit time when the stock level is x. Backlogging of demands is permitted when $s(t) \leqq 0$ and in this case $\ell(x(t))$ represents the penalty for delay in satisfying demands. For example, we might have $\ell(x) = hx$ when $x \geqq 0$, $\ell(x) = -px$ when $x \leqq 0$, where the constants $h, p > 0$ represent the holding cost per unit quantity stored and the penalty per unit shortage respectively.

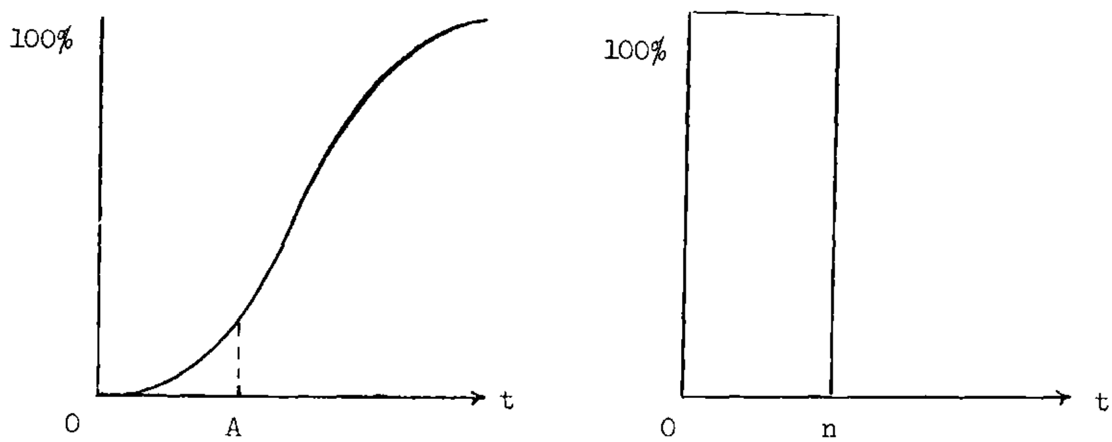


Figure 1. Generalized Time-Loss Functions

Bather does not consider actual product price since demands may be backlogged and eventually satisfied. His optimization problem then " . . . is to find the operating policy which maintains the best compromise between too many orders and frequent set-up costs on the one hand, and excessive stockpiling or backlogging on the other." Note the direct relationship between quantity stored and holding cost with no provision for product depreciation or total loss due to perishability. Seasonality may be provided for in his model since demands are allowed to vary continuously.

In response to increased treatment of the inventory control problem in the time domain as a dynamic system, several articles have appeared suggesting a systems approach to inventory. One such article by Reed and Hanken (25) proposed a means of viewing inventory and the inventory problem. They reason that this " . . . approach attempts to consider the broad consequences upon the entire system of changing one part." Viewing the problem from a similar approach Alcalay and Buffa (1) propose a general model of a production system. They recognize this as a subsystem of the total organization system having the goal of maintaining stability while advancing by optimal degrees toward a prescribed goal. Their purpose was to present a conceptual framework toward the development of a mathematical model of a production system, not unlike Zangwill's paper previously mentioned (37), nor this research itself. The difference is mainly in defining the particular system in question and the mathematical techniques employed.

More recently Buffa, allied with Reissman (26) developed an even more elaborate structural and mathematical framework for the analysis or synthesis of a large class of production systems. The general applicability of the model is demonstrated by comparing it with the industrially validated model developed by Forrester (16). Suggesting that Forrester's model is indeed a very special case of their own, they profess their model to have a high degree of general conceptual utility.

Industrial Dynamics

Since the methodology has evolved now to the point of clearly conceptualizing the problem as a principal initial step in industrial systems optimization techniques, it is well to include industrial dynamics as a significant separate contribution to the field. "Industrial dynamics is a way of studying the behavior of industrial systems to show how policies, decisions. structure, and delays are interrelated to influence growth and stability" (16). Forrester cautions that for " . . . most of today's great management problems, mathematical methods fall far short of being able to find the 'best' solution." The same attitude is reflected by Reed and Hanken (25) when they encourage the use of the systems approach "to assure that the optimum sought is the true optimum and not just a suboptimum." Postulating that in the future there will be less concern with actual operating decisions and more with the policy basis that should guide operating decisions, Forrester presents a method for modeling industrial systems based on more descriptive information than statistics and formal data. He points out that this type model should be developed first and used to determine what formal data need be taken.

Because of the conspicuous absence in the literature of control models dealing directly with perishable inventory and the obviously vast application of such models, it is surmised that an industrial dynamics-type model is needed. An extension of the classical retail store model developed by Forrester to include perishable inventory appears to be an ideal method for studying the problem in question. Not only does this technique avoid traditional mathematical preoccupance with precision, but also provides an excellent means of viewing the system and its reaction to changing policies by computer simulation.

CHAPTER III

OBJECTIVES AND APPROACH

Objectives

The purpose of this research is to develop a means of conceptualizing the internal causal relationships of a perishable, seasonal inventory control system. Its objectives are:

(1) To identify those information feedback loops which control the patterns of behavior of this system.

(2) To model the system, availing these feedback loops as a mechanism for policy formulation.

(3) To simulate the system, using the developed model in order to analyze the desirability of introducing policy changes.

Scope

The general nature of the problem under analysis is discussed in Chapter I. Actual conditions, policies, and appropriate data used in the model were collected from relevant businesses in Atlanta, Georgia. Two businesses in particular were selected as a basis for modeling because of the perishable nature of their product (flowers) and the established stability of their operating policies (Appendix A). Although it is recognized that floral products are highly perishable and often seasonal, the model appears to have equal application to interests as diverse as ammunition and dairy products.

Approach

The following procedure is cited as the means of applying the methodology of Industrial Dynamics to enterprise design:

(1) Describe the problem and select the possible patterns of behavior of the system's principal variables.

(2) Define the system environment by determining its boundaries, and identify the main factors included.

(3) Identify the feedback loops responsible for the system's internal causal relationships and general behavioral pattern.

(4) Construct a mathematical model of these interactions.

(5) Generate the behavior of the model through time.

(6) Compare (5) with (1) and redesign/revise the model until it is an acceptable representation of the real system.

(7) Select appropriate targets for improvement and redesign/revise (6) in such a way as to guide the model's behavior toward those targets (16,30).

Because of the continuous nature of this methodology the internal system relationships must be self-adjusting, adaptive to changes both within and outside the system's environment. Information feedback loops provide the vehicle for effecting this internal adjustment.

Basically, an information feedback loop consists of an accumulation, flow rate, and information (30). The flow rate causes a time variant accumulation. Information concerning the accumulation is compared with the desired accumulation status, and the flow rate is adjusted appropriately through a decision making process. The loop is completed when the adjusted flow rate causes a change in accumulation

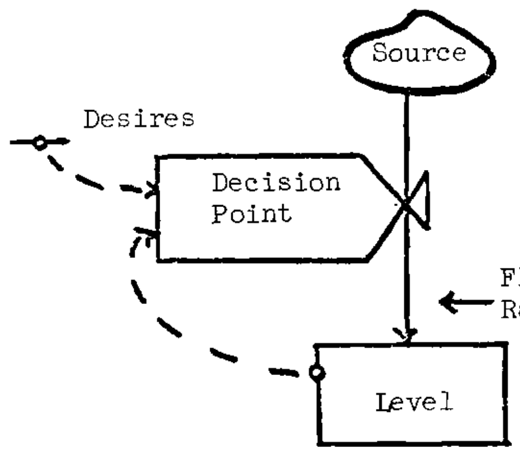


Figure 2. Basic Feedback Loop.

(called LEVEL).

Information feedback loops may be either positive or negative. If, given the information that the accumulation is increasing (decreasing), the flow rate is adjusted to cause the accumulation to increase (decrease) even more, the loop is positive. However, if under the same conditions the flow rate is adjusted to cause the accumulation to decrease (increase), the loop is negative. The positive loop is related to growth while the negative loop to goal-seeking (30).

The focus of the feedback loop is the decision point. Herein continuous adjustments are effected which ultimately cause the system's observed behavior.

> Decisions fundamentally involve three things. First is the creation of a concept of a desired state of affairs.... Second, there is the apparent state of actual conditions.... The third part of the decision process is the generation of the kinds of action that will be taken in accordance with any discrepancy which can be detected between the apparent and desired conditions. (16)

The relationship between the first two and the resultant action to be taken is referred to in Industrial Dynamics as policy. Conceptually the same as a decision rule or transfer function, policy in the basic information feedback loop discussed is that phenomenon which describes the flow rate change given some discrepancy between desired and actual level accumulation.

CHAPTER IV

DESCRIPTION OF THE PROBLEM

The initial step in Industrial Dynamics methodology is to describe the system that creates the problems which motivated the study and possible causes of those problems. The inventory system under analysis is described in Chapter I, and in general the significant problems involved in that system are threefold:

(1) Perishability

(2) Seasonality

(3) Backordering

Perishability

As has been suggested perishability of any given product might well be represented through time as a convex function described in Figure 1 . Moreover the step function treatment of perishability used by Bulinshaya is actually employed as an operating policy at Borg's Florists'. That is, all product remaining at the end of some operating period is lost. The step function may be considered a special case of a convex function, its unique characteristics in this case being the result of retarding perishability in a refrigerated room. Hence, a convex function, which may be uniquely defined for any given product, will be used here to describe product perishability.

The question of perishability retardation might be worthy of consideration as a distinctly different problem because of the inherent function involved. The increased cost associated with increasing refrigeration capacity, adding additional greenhouses, installing additional airconditioning units and so forth may well be approximated by a concave cost function as the size of inventory accumulation increases. However, since such environmental changes can hardly be considered continuous, especially when being considered in relation to the time domain within which the products under analysis perish, perishability retardation costs will be considered constant, subject to incremental changes given a managerial decision to assume some additional fixed cost to that end.

To be consistent it is assumed, therefore, that there exists a capacity limitation on inventory accumulation, be it cubic feet in a refrigerated room or number of greenhouses available. However, even under these conditions perishability retardation and/or storage costs will vary with the size of inventory accumulation, as it costs more to cool a filled refrigerated room than an empty one. For the purposes of this study the variance will be assumed to be negligible, since the particular system being modeled is essentially stable and does not realize violent inventory fluctuation.

Seasonality

The problem of seasonality is essentially a problem of consumer demand. The effect may be produced by introducing some stimulus into an otherwise stable, possibly uniform, demand function. Since the function is seasonal, the location in time as to where the stimulus is introduced is fairly predictable. However, such is not the case with respect to the effect of the stimulus on the various interacting components of a given system.

Seasonal demand functions may be approximated in a variety of ways. For example, demand for rose bushes might have a normal distribution spread over the late winter and spring months. Ammunition might have a step demand function, rising and dropping at the beginning and end of hunting season. The season itself might further define the nature of the function. Mothers' Day and Easter are examples of stimulants effecting the demand function of a product such as carnations that otherwise might have a stable, nearly uniform, distribution. In this case the demand function might be described by a truncated normal distribution superimposed on a uniform distribution, demand rising normally out of uniformity as the season approaches and dropping sharply back to uniformity as it passes.

Clearly demand functions in the real world are in fact a variety of combinations of trends and oscillations, and not well behaved distributions. Yet, by approximating demand functions one may better identify certain patterns of systems behavior. And, more important, if an analysis of these patterns indicates that a more thorough description of the demand function is necessary, the system manager may confidently allocate necessary resources to that end.

The seasonal demand function is often subjected to distorting influences. Advertising and fluctuating prices are two such influences. Although this research does not provide for these managerial alternatives, they are mentioned here to point out still another facet of interacting forces bearing on the system. There is little question that advertising and price variance have considerable potential for influencing the system's behavioral patterns. The manager must be aware of this potential and his related policies in a competitive market. As discussed in Appendix A neither Borg's Florists' nor Smith's Nurseries utilize these techniques.

By integrating the problem of perishability and seasonality the inventory manager is confronted with an even more complex problem. To satisfy seasonal demands he must maintain a variable size inventory. Yet his inventory must be minimized in order to minimize perishability loss. Obviously too little inventory, while minimizing perishability loss, may lead to lost sales. In the case of the system being modeled perishability loss is preferred to lost sales. Hence the problem is to minimize perishability loss while simultaneously minimizing lost sales.

Backorders

The third significant problem in the system being analyzed involves backorders. In general backordering allows the manager to accept orders for goods without having those goods immediately available. This operating policy relieves the manager of the requirement of maintaining an associated amount of on-hand inventory with its related storage cost and perishability loss. At the same time consumers, who would otherwise be lost sales, may be satisfied to backorder. The manager is now faced with the problem of how much inventory may be backordered without the inherent problems of administration, delay, shipping, receiving, and possibly even cancellations.

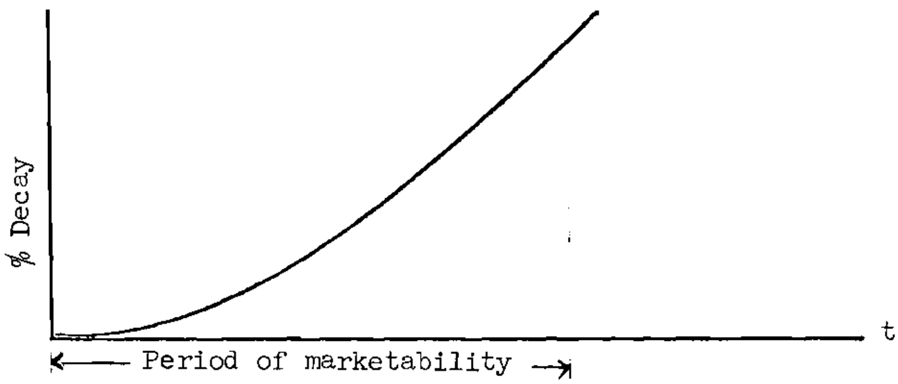
Backordering appears highly attractive when dealing with the problem of seasonal goods. Orders may be accepted well in advance in anticipation of the season. In many cases a lower price may be offered to attract such orders. Although the particular problem of price variation will not be handled directly in this study, its influence must be considered. As mentioned earlier there is a capacity limitation on inventory storage. The manager may allocate all of this capacity to handling backorders as they become due. By doing so he would have no goods on-hand to satisfy immediate orders at currently higher prices. Obviously the manager would prefer to satisfy those immediate orders, avoid lost sales, gain the related in-season profit, and handle backorders to the greatest extent possible. In this model as in the case of Smith's Nursery some optimum balance will be sought with priority to filling backorders to which the system is commited at the risk of lost sales regardless of price differential.

Production

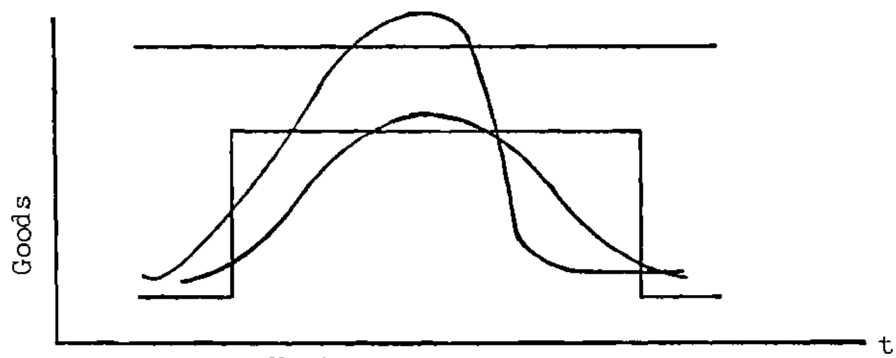
The manager who has the additional responsibility of production is faced with a still more complex problem. The product must not be produced/procured too early lest it perish before the season. Moreover there are economic advantages in producing/procuring seasonal inventory early such as receiving a reduced price at wholesale or using the production facility for still another product. Except to a very limited extent in the case of Smith's Nursery, the system under analysis controls production/procurement by contracting with outside sources and is therefore not significantly concerned with production starting time, plant capacity, or resource allocation to production. The manager is more concerned with the dynamics of the purchase order quantity submitted to the producer. Nonetheless, the problem exists, and it must be provided for when dealing with the environmentally broader problem of a production-inventory control system.

Behavior Patterns

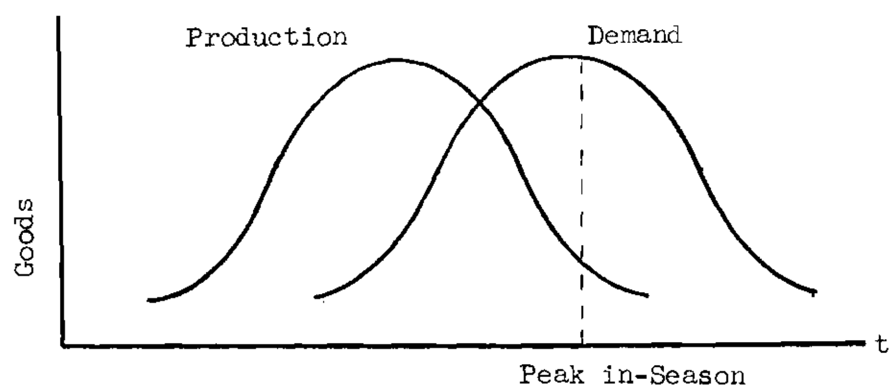
The following patterns of behavior are assumed to be representative of perishability and various types of seasonal demand. The assumed relationship between rates of production and seasonal demands is also portrayed. The constructed model should have patterns which behave in a similar fashion. However, that in itself is not sufficient to conclude that the model adequately represents the real system. Also necessary is the subjective belief that the facts included in the model represent the most significant factors in the system (30).



Perishability Function



Various Demand Functions



Production vs Current Seasonal Demand Relationship

Figure 3. Hypothesis of Behavior.

CHAPTER V

THE MODEL

After establishing the most significant problems in the system, the next step in Industrial Dynamics methodology is to define the boundaries of the system. One technique of accomplishing this is to identify the sectors of behavior within the system. These sectors should have the characteristic that when coupled or perceived jointly they make up the system itself and define its boundaries. Additionally, they should provide a means of identifying the feedback loops which determine the system's behavioral patterns.

Sectors of Behavior

The system under analysis may be devided into four major sectors: Retail, Wholesale, Market, and Perishability. The wholesale sector represents the availability of goods to the retailer. It is often represented physically by a production facility, and in the case of the system being modeled it is one or a series of controlled environment farming stations. The wholesaler or producer receives orders from the retailer and in turn, after processing those orders, ships the desired goods to retail. Because the receipt of purchase orders at wholesale represents an accumulation, there is an inherent delay associated with this process. Production control is achieved by controlling the rate of flow of information and goods into and out of wholesale. Although such control is not the objective of this study,

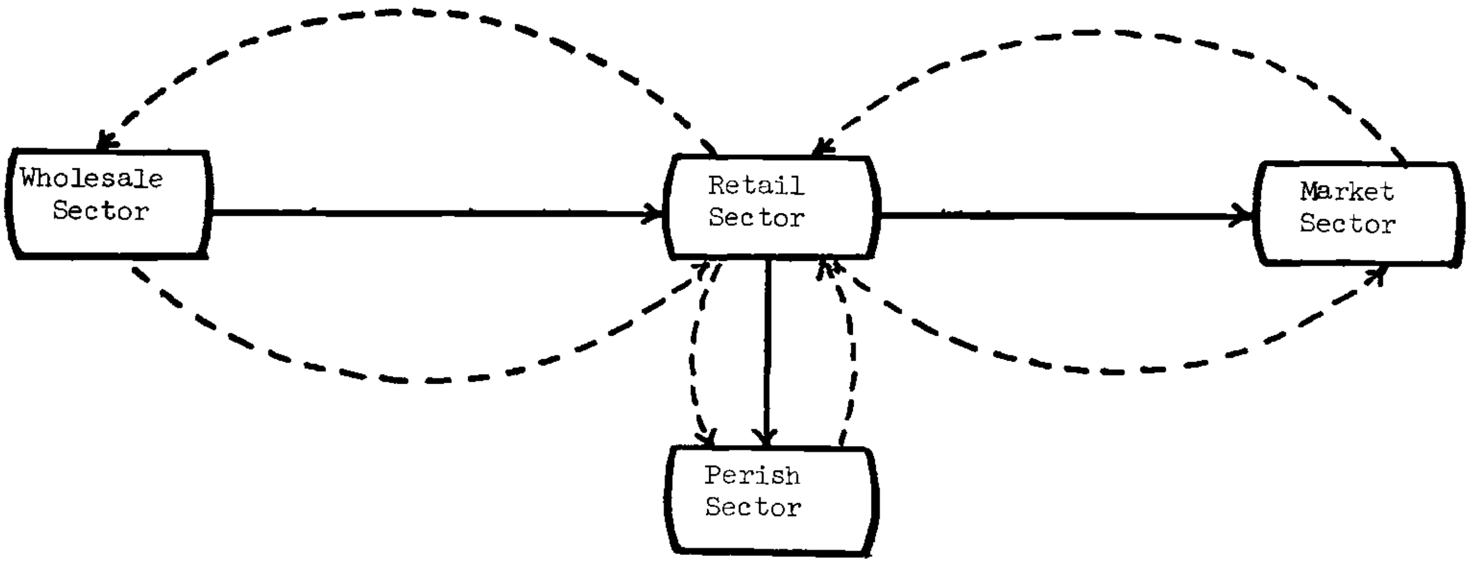


Figure 4. System Environment.

it is mentioned here to point out the relationship between flows and accumulations in policy formulation. The wholesaler may seek control by such methods as contracting for some minimum/maximum purchase order rate from the retailer, seeking out that number of retailers for contract such that his purchase order rate is operationally feasible, or in a related manner advertising his services to the extent he deems necessary to attract adequate purchase orders. On the other hand he controls goods' flow by controlling production rate. This may be achieved by varying employment level (another accumulation with related flows), capital expenditure in the form of equipment, buildings, etcetera, or incorporating associated wholesalers. Still another means of controlling goods' flow is to control the mode of transport from wholesale to retail. Another tool available to the producer for affecting the internal interactions of his sector in this system is an information flow to the retailer. Such information may consist of special prices, or commodities available due to some fluctuation in or related to the wholesale sector. Should the producer realize some violent oscillations within his sector it becomes obvious that those same oscillations may be transmitted to, and adversely affect, the retailer unless adequate control measures are pursued by the latter. These control measures and other information concerning the links between wholesale and retail - purchase orders, goods flow, and other information - will be discussed later when analyzing the specific problems of the internal relationship of the system being modeled.

The market sector represents the consumer in this system. The consumer receives goods from the retailer. The rate at which goods are shipped to the consumer is based upon the demand rate, or the requisitions received at retail from the consumer, and the availability of goods at retail. If more requisitions are received than there are goods available, the goods are backordered. Otherwise requisitions are filled immediately from inventory. Although retail may often influence the rate of demand by advertising and/or price variation, this system does not include those alternatives. Feedback information from retail to market is in the form of good service at retail, minimum lost sales and good quality merchandise. Retail depends upon these good business practices to maintain existing customers and attract new ones. The particular system being modeled has established its stability over an extended period of years. It is therefore assumed that the policy of the cited business practices is successful, and the market is essentially stable. Stability here is not meant to infer a constant demand function, but rather a uniform function with some variability and fairly predictable seasonal patterns. For these reasons demand will be handled as a system input, independent of the feedback from retail to market in the sense that this beedback causes a stable market reaction and does not produce violent or unpredictable oscillations or trends in the market demand function.

The perishability loss sector of the system is similar to the market sector in that it too represents a reduction of goods at retail. Its reduction function, analogous to the market demand function, is approximated by the convex perishability function discussed earlier. Goods arriving at retail from wholesale are immediately subjected to the effects of this sector of the system just as they are to those of the market sector. As the market demands are satisfied those goods remaining at retail become less marketable because of their perishable nature. The system manager's decision as to the degree of quality he requires of his goods determines the length of the period of marketability, that is the maximum period over which any given item may be subjected to perishability (held in storage) and still be used to satisfy market demand. The rate at which goods perish is dependent upon the convex perishability function and the length of time goods are available in inventory. That delay of goods in inventory varies continuously with the perishability rate, the rate shipments are sent from retail, and the amount of inventory on-hand. The accumulation of perishable goods is controlled/minimized by controlling these rates. Clearly perishability accumulation will be a monotonically non-decreasing function. One goal of the manager then is to minimize the slope of this function.

The nucleus of this system is the retail sector. Requisitions are received at retail from the market sector, and in return goods are shipped back to market. The difference between potential and actual roles represents lost sales which must be minimized. The relationship between potential and actual sales will be discussed later. Upon receipt of requisitions the retail sector prepares and submits purchase sales requests to wholesale. As in the case of every accumulation there is a delay associated with the accumulation of requisitions at retail. This accumulation may be controlled by the requisition rate and the purchase sales rate. However, since the requisition rate is assumed to be an independent variable input, the manager may control requisition accumulation only via the rate at which he processes purchase sales requests to wholesale. In response to the purchase sales requests the retail sector receives shipments of goods from wholesale after an associated inherent delay. The model assumes that the retailer has identified a stable wholesale sector, negating the unfavorable effect of uncontrollably oscillating goods' availability. Goods accumulate at retail and are shipped to market in response to the level of accumulation of unfilled orders. Control of the inventory accumulation at this point is critical, since the greater the accumulation the more significant the effect of perishability, and conversely the smaller the accumulation the greater the possibility of lost sales. A certain amount of goods perishes, and that amount accumulates in a momotonically non-decreasing manner. Information concerning the rate of accumulation of perished goods is fedback to the retail sector, and the inventory accumulation is reduced appropriately. Hence, the manager, in order to control inventory accumulation, must control not only goods' shipping and receiving rates but also the rate at which perished goods accumulate. This model assumes that control of perishability rate may not be accomplished continuously, and only in the case of a dominant effect on the system will the perishability rate be artifically controlled by a means such as refrigeration. Otherwise continuous control of the rate at which perished goods accumulate is attempted by minimizing the amount and length of time inventory is on-hand, thereby controlling the rate at which goods are available to perish.

Feedback Loops

After establishing the main sectors in the system's environment the next step in the methodology is to identify the feedback loops responsible for the system's behavioral patterns. Recalling that feedback loops are comprised of accumulations, flow rates, and auxiliaries, an additional characteristic must now be considered. This is essentially the force which stimulates the causal adjustment within the loop. As an accumulation of inventory (for example) builds up or is reduced substantially the control manager may be exposed to some force or pressure to adjust his inventory to a "better" condition. Forces may be in the form of excess storage cost, too little inventory to satisfy orders, and so forth. The manager adjusts the appropriate flow rates, information regarding the results of this adjustment is fedback, new pressures are introduced, and the process is renewed. The forces stimulating action in the system under analysis will be identified in this discussion of the significant system feedback loops.

The forcing function in this system originates in the market sector. For a variety of reasons pressures for more goods build up in the market, which may be thought of as an accumulation of this system's end product. Those pressures manifest themselves as a variable demand upon the retail sector. Demands accumulate in the form of an unfilled order rate. All orders are considered unfilled initially, though the waiting period may range from almost immediate fill to several days. The accumulation of unfilled orders introduces a pressure upon the manager to satisfy these demands. His alternatives are to fill the orders with inventory available at retail, backorder the requisition, or refuse the requisition. Since his policy is to minimize the latter alternative he is ultimately faced with the problem of submitting purchase sales requests to wholesale for additional goods in order to satisfy demand and/or replace stock. A related pressure is his awareness of market conditions as various seasons approach, and he must submit advance purchase sales requests to wholesale in order to have goods available for that anticipated season (Figure 5).

The pressures resulting from an accumulation of purchase sales requests at wholesale forces the producer to action. Our interest in this study is principally in the results of that action as pertains to the rate of shipment of goods received at retail. It is recognized as a weakness of this model that the cause and effect interactions within the wholesale sector are not included. Changing production rate, employment levels, capital equipment, raw materials, variable production completion times, and related pressures and forces are all relevant and extremely important to the dynamics of this system. Nonetheless it is possible to learn something of the behavior of the perishable, seasonal control system by assuming that the production/ procurement facility included in the wholesale sector is stable. Therefore, only the rate at which shipments are received at retail resulting from the purchase sales requests from retail are of interest.

As shipments are received at retail pressures in the form of excess storage costs and limited storage capacity prompt the system manager to action. The obvious point of relief of these pressures is the shipment of goods to market, satisfying demands and closing the loop. It is the rate of shipments sent from retail with which we are

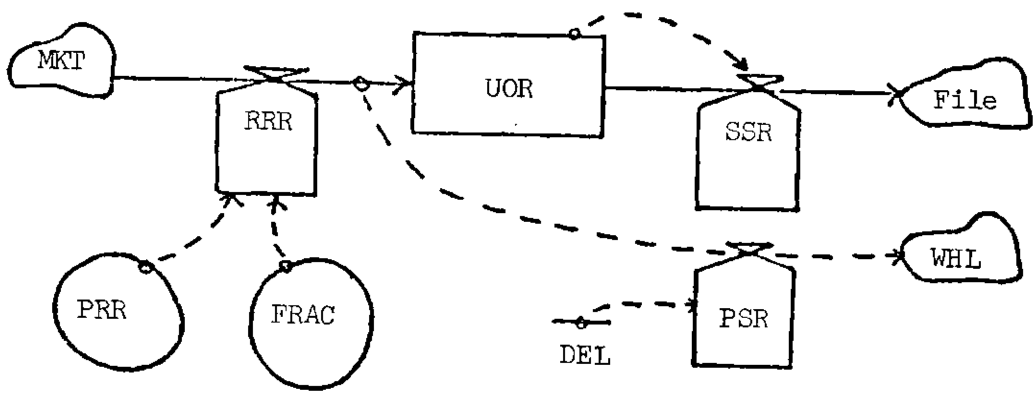


Figure 5. Basic Information Flow.

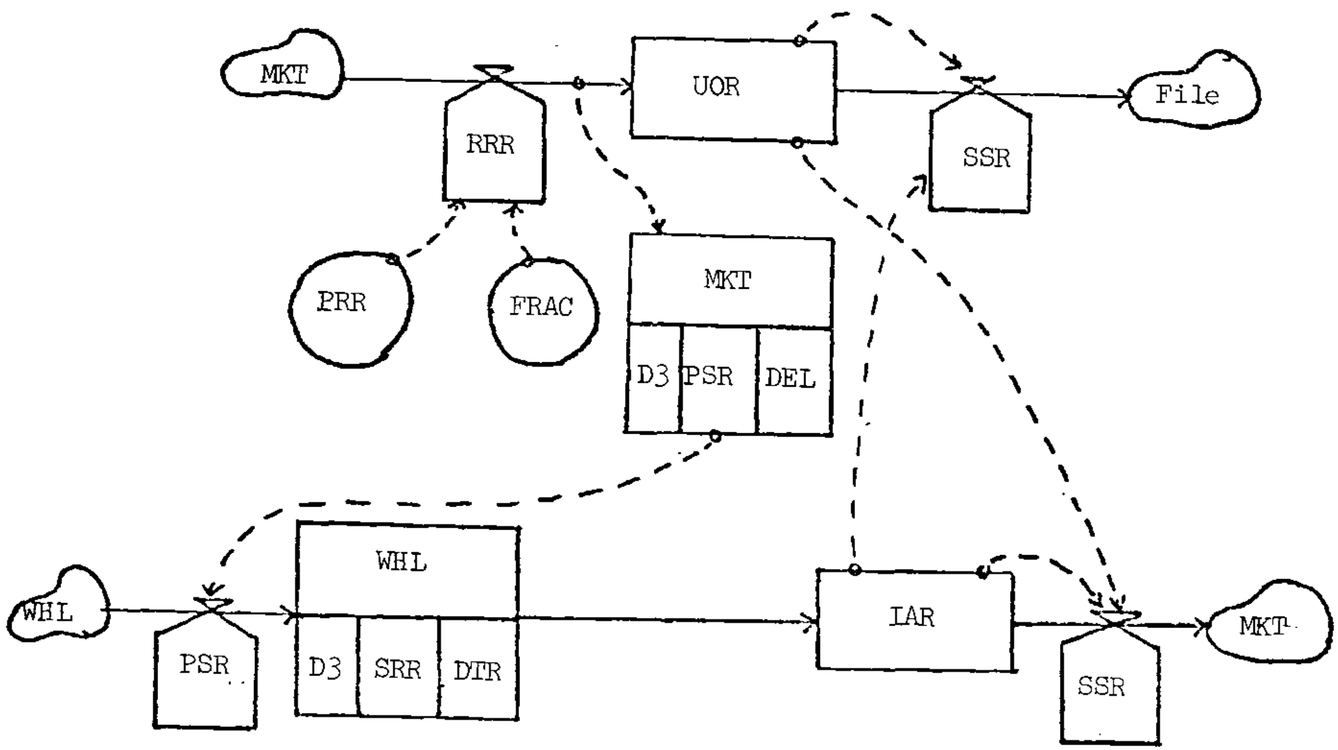


Figure 6. Basic Information and Goods Flow.

interested. A related alternative to relieving the pressures resulting from the size of inventory accumulation at retail is to stimulate additional demands through advertising. Since this system relies on product quality and service for its advertisement, that alternative is assumed to be included in shipping goods to market. Hence the manager may relieve the forces associated with inventory accumulation by:

(1) Ship goods to market.

(2) Backorder.

(3) Lost Sales.

A negative effect of the pressures from inventory accumulation is manifest in perishability loss. All inventory at retail is subject to perishing, and the longer inventory is maintained on-hand the greater the potential loss. The length of time inventory is delayed at retail is a function of the amount of inventory on-hand and the average rates of flow out of retail both to market and to perishability loss. The perishability rate then varies continuously with that delay, or availability period, at retail, and the perishability function is uniquely defined for any given product. From the point of view of minimizing perishability loss it appears essential that the delay period of goods at retail must be minimized. Yet the shorter the availability period the less opportunity the retailer has to sell his product, thereby increasing the probability of lost sales. Percent of sales lost is a measure of the difference between potential and actual sales, the latter being a function of both the former and the amount of time the product is exposed to the market. As the exposure or availability period is extended actual sales closely approximates potential sales, and lost sales are minimized. A flow diagram of these relationships is illustrated in Figure 7.

System Variables

As is the case of any system, the behavior of certain variables within the perishable, seasonal inventory control system determines its resultant output. It is often difficult for the system manager to evaluate the relative significance of each of these variables, and in all likelyhood that significance will fluctuate as the season varies. More important the manager may frequently have to deal with variables over which he has only limited control. In this system these variables of limited control establish the basis upon which the manager formulates operating decisions by manipulating the system variables within his direct control. The following is a discussion of the variables deemed significant in this system.

(1) Limited Control.

The first variable to be discussed deals directly with perishability. Every product with this characteristic has a uniquely defined perishability function. For the purpose of this study that function will behave generally as shown in Figure 1. Specifically the function used in the mathematical model simulation is illustrated in Figure 8.

This function may be described in DYNAMO simulation computer language as follows:

```
FRA.K = TABHL (DEC, AVDL.K, 0, 14, 1)
DEC* = 0/1/3/7/16/25/38/50/62/75/84/93/97/99/100
```

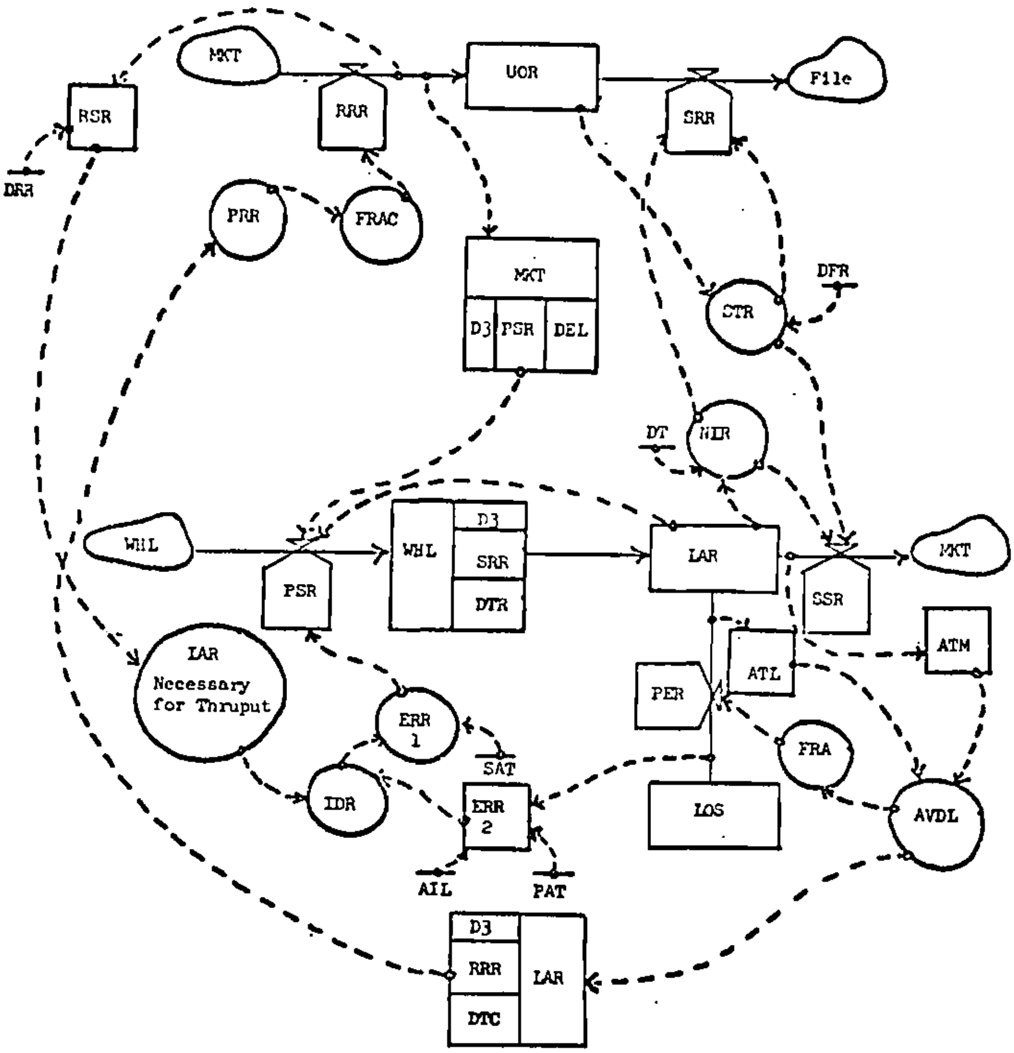


Figure 7. System Model.

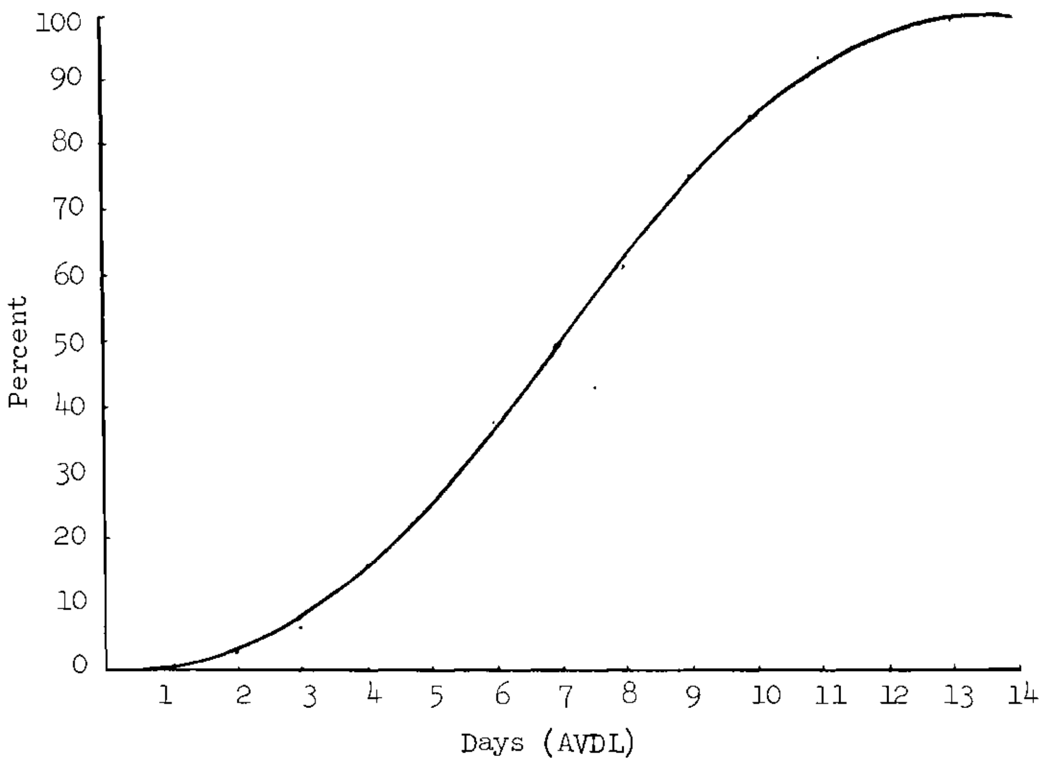


Figure 8. Perishability Function

These equations represent fifteen sets of data points which define an arbitrary perishability function. As the average delay of goods in inventory varies the fractional amounts of perishability varies so that after one day the on-hand inventory has perished a total of 1 per cent, two days 3 per cent, and so forth. This particular function is defined purely for the academic purpose of analyzing the impact of perishability on an inventory control system. In order to adapt the model to his particular system a manager need only replace the assumed data with his particular data points of per cent perishability over time. The limited control a manager may exercise over this variable is manifest in techniques such as refrigeration, handling, and packaging. Control is limited in the sense that eventually the product will perish. Nonetheless, that eventually may be retarded and the shape of the perishability function altered by a number of procedural innovations such as those cited. That alternative might be costly however, and the manager must be able to measure as closely as possible the value of such a move.

Another variable over which the system manager exercises only limited control is sales rate. Sales rate is considered here to be a function of the number of demands made upon the retail sector by the market sector and the availability of goods in inventory. As has already been discussed, the market demand, or potential sales rate, is assumed to be an independent varible input. Hence, the manager may exercise limited control over actual sales rate only via the relationship between product availability at retail and its effect upon the market. Again as an academic tool for modeling purposes a general functional relationship between potential sales and product availability is assumed to exist as illustrated in Figure 9. This function may be described in DYNAMO simulation computer language as follows:

```
FRA.K = TABHL (SAL, AVDL.K, 0, 5, 1)
SAL* = 0/50/78/90/98/99/100
```

If the product is available for sale in inventory for one day there is a 50 per cent probability that a potential sale will be satisfied, two days, 78 per cent, and so forth. As in the case of the perishability function, the intent here is not to define a particular availability/probability relationship, but merely to establish the existance of such a relationship and its significance to the system. Each product will have a unique functional relationship, and the manager may exercise limited control over the function's pattern by advertising and/or hiring additional sales personnel, as well as extending the product's availability period as in the case of the perishability function. The manager's actual sales rate is a portion of the potential sales rate, that portion defined by the sales probability function; and his lost sales is a measure of the difference between potential and actual sales.

Probably the most significant variable over which the manager has only limited control is the average amount of time that perishable goods are available on-hand in inventory, hereafter referred to as the average delay. The average delay is a function of the current level of on-hand inventory and the rate at which that level is depleted both by market consumption and perishability. In this model the market

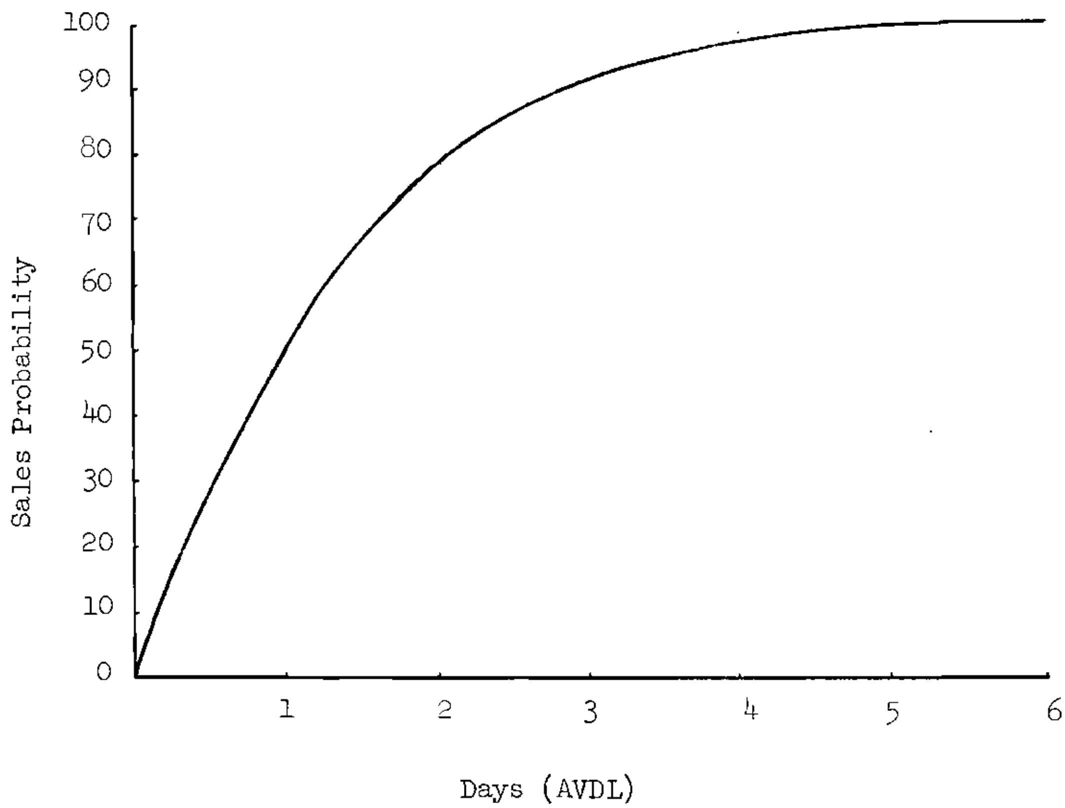


Figure 9. Sales Probability Function.

consumption rate will be that rate at which all unfilled orders are filled or inventory level is depleted whichever occurs first. No. control mechanism in the form of backorder accumulation and related flows is provided for the manager. Additionally artificial control of perishability (refrigeration) will be deferred in favor of more imaginitive managerial efforts, while such constructive efforts are yet available. Therefore, the manager has only the current on-hand inventory level available to him as a means of controlling its average delay at retail. Since that level is also a function of the rate at which goods are received at retail, control of the latter flow rate is the manager's chief means of controlling average delay. In this model the rate at which goods are received at retail is directly proportional to the rate at which the manager requests goods from wholesale. Hence his control of average delay is ultimately achieved through his control of the purchase sales request rate and the variables and parameters included therein.

(2) Direct Control.

A number of parameters in the perishable, seasonal inventory control system allow the manager a means of direct control over his system's output. At this point those parameters, all of which are time related, will be identified and discussed briefly. An evaluation of the sensitivity of the model to their variance will be deferred until the complete model of simulation equations is constructed and its output analyzed.

Several of the direct control parameters under consideration are referred to in Industrial Dynamics as system's delays. The significant delays in this system include but are not limited to:

(a) the delay in filling orders at retail.

(b) the delay in sending purchase sales requests to wholesale in order to refill depleted inventory.

(c) the delay associated with the customer's becoming aware of product availability at retail.

(d) the delay of goods in transit from wholesale to retail.

(e) the delay associated with adjusting inventory based on the error existing between actual and desired inventory level.

(f) the delay associated with adjusting inventory based on the product perishability rate.

Other time related direct control parameters include:

(a) the averaging time constant used to compute the average rate of flow of goods to market and to perishability loss.

(b) the time constant used for smoothing requisition or actual sales receipt rate.

(c) the averaging time constant used to compute the long range actual and potential sales rate.

(d) the constant used to determinize the desired amount of inventory to have on-hand continuously over same fixed predetermined period of time.

With the possible exception of in transit delay of goods from wholesale to retail the manager may exercise considerable direct control over his system's output through these parameters and variables. Moreover the transit delay too may be directly controlled by the manager by his requiring under contract a certain degree of responsiveness of wholesale to his needs. As will be seen later the manager may even be willing to accept a more expensive contract depending upon the impact of this delay on his system.

System Costs

Inherent in most inventory control problems are cost considerations with which the manager must deal continuously. Although a broad variety of costs are involved in this system only those which are unique to perishable, seasonal inventory will be discussed. Moreover, these particular costs, namely perishability and lost sales, are unique to the system chiefly in their mutually opposing relationship, which may be balanced by proper control of the average delay.

During the course of normal operations a manager may realize a varying amount of cost resulting from perishability. That cost fluctuates with perishability rate, and is therefore greater the longer the product is on-hand at retail. A typical cost function for perishability is illustrated in Figure 10. Notice that this figure, plotted against time, portrays the range over which a manager may expect his product to be on-hand on the average. The cost varies between X' and Y' given this particular set of conditions. The perishability cost function typifies a means by which a manager may analyze his own system simply by projecting in this way appropriate information on a given product. Although different products will understandably have different associated values for average delay and cost ranges, an appreciation for the inter-relationship between these two variables is stressed here, providing for a fuller and more general understanding of the dynamics involved in the managerial control of perishable, seasonal inventory.

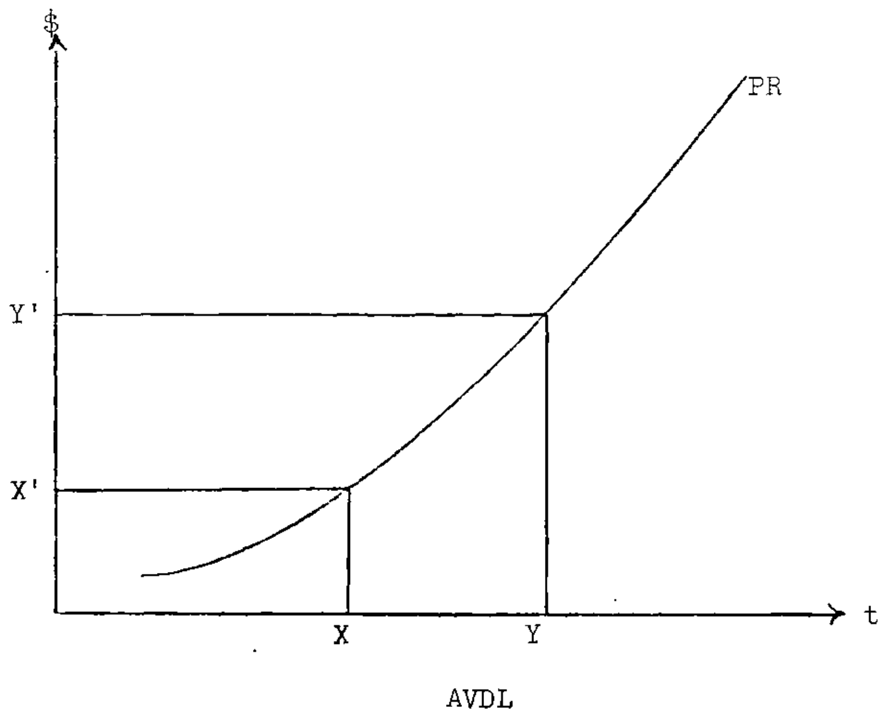


Figure 10. Perishability Cost Function.

A similar cost consideration is manifest in the form of lost sales. As was discussed earlier as the average time that goods are available for sale at retail is extended, the probability of consumating that sale is improved, or the probability of a lost sale is reduced. By quantitatively relating a cost value to the per cent of lost sales it becomes apparent that as he reduces his probability of lost sales by extending the average delay the system manager may likewise reduce the cost associated with lost sales. This relationship is illustrated in Figure 11. In a manner analogous to the perishability cost function lost sales costs vary inversely with average delay. Now the manager is burdened with limiting constraints wherein he may not reduce his average delay indiscriminately because of the lost sales cost. Similarly by extending his average delay to reduce lost sales the manager generates an increased cost associated with perishability loss. Clearly there may exist some best range over which the average delay should vary to minimize both perishability as well as lost sales costs.

By combining the two cost functions and summing them to find the total cost function the average delay range for minimizing total cost becomes apparent. Additionally the effect on perishability cost and lost sales cost resulting from adjusting the system to this new average delay may be observed. These relationships are illustrated in Figure 12. By adopting the policy of delaying goods in inventory on the average between X and Y days, the cost of lost sales varies between A and B and the cost of perishability loss between C and D. Assuming, however, that the dynamics of the system currently provide for an average delay between X' and Y' (Figure 13), the manager is now faced with the

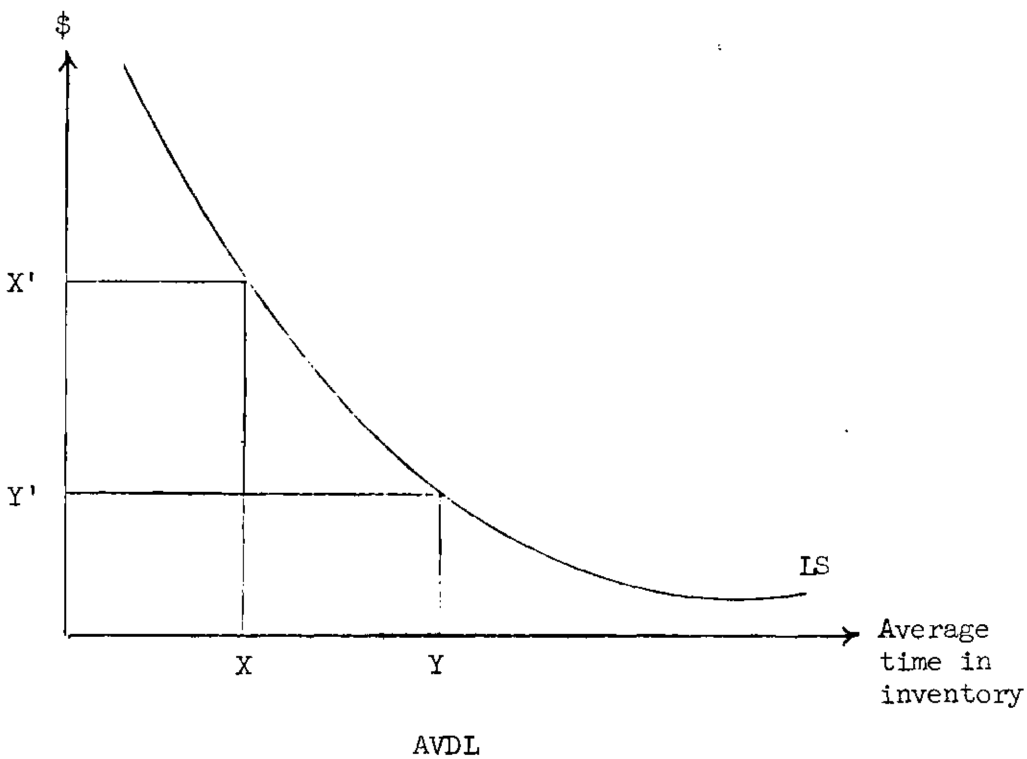


Figure 11. Lost Sales Cost Function.

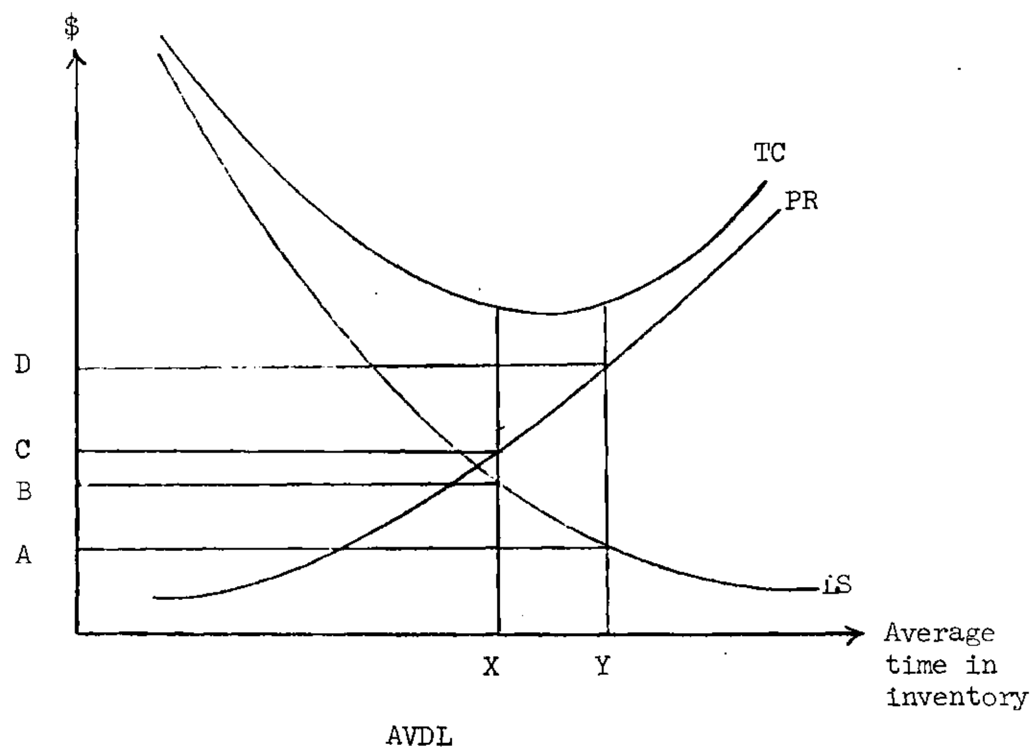


Figure 12. Minimizing Total Costs

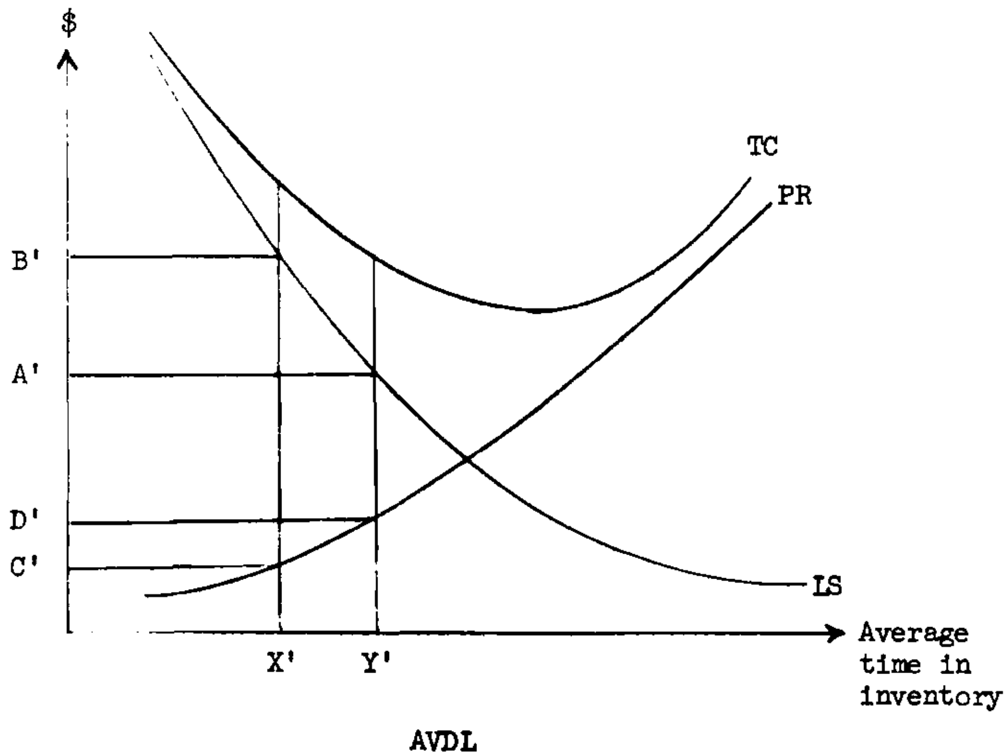


Figure 13. Undesirable TC/AVDL Combination

problem of influencing those dynamics so as to adjust his average delay and minimize total cost. In the discussion of system variables over which the manager has only limited control it was pointed out that the average delay is the principal means of controlling costs in this model and may be influenced only via the rate at which purchase requests, including continuous adjustment for error, are sent to wholesale. As will be demonstrated in the next chapter a system adjustment of this nature requires an in-depth understanding of the internal causal relationships within the feedback loops of the system under analysis.

System Equation

When writing the equations for a system simulated in DYNAMO computer language, the concept of time is extremely significant. Past, present and future actions, designated J, K and L respectively are separated by an increment of time, DT, the length of which is based primarily on the degree of accuracy the model designer seeks. The intervals between each of these periods are designated JK and KL respectively and represent passage from past to present and present to future.

Although many of the equations that follow and their interpretations may be found in Chapter 15, Industrial Dynamics, they are included here for clarity and continuity.

The first equation of interest in this model represents the level of inventory of perishable goods currently at retail.

$$IAR.K = IAR.J + (DT)(SRR.JK\text{-}SSR.JK)$$

IAR = Inventory Actually at Retail (goods)

SRR = Shipment Received at Retail (goods/day)

SSR = Shipments Sent from Retail (goods/day)

DT = Delta Time (days)

Physically this equation means that the current level of inventory at retail equals yesterday's level plus the difference between the amount of goods shipped from and received at retail between yesterday and today. Similarly the level of unfilled orders at retail may be expressed as follows:

$$UOR.K = UOK.J + (DT)\ (RRR.JK\text{-}SSR.JK)$$

UOR = Unfilled Orders at Retail (goods)

RRR = Actual sales Rate Received at Retail (goods/day)

The rate at which shipments may be sent from retail is dependent upon the level of inventory and number of unfilled orders. The maximum amount that may be sent to market without the inventory becoming negative is:

$$NIR.K = IAR.K/DT$$

NIR = Negative Inventory at Retail (goods/day)

Moreover the number of goods shipped cannot exceed the demand. Hence the number of shipments from retail attempted must be less than or equal to the number of unfilled orders.

$$STR.K = UOR.K/DFR$$

STR = Shipments Tried from Retail (goods/day)

DFR = Delay in Filling orders at Retail (days)

The actual shipment rate then is the minimum of those two conditions.

$$SSR.KL = MIN\ (STR.K,\ NIR.K)$$

SSR = Shipment Sent from Retail (goods/day)

The rate at which sales requests are sent from retail to wholesale is determined by the rate at which actual sales are consumated and the manager's decision concerning how long to delay requesting replacement stock after a sale. In this model the sales request rate to wholesale is approximated by a third order delay as discussed in Forrester's Industrial Dynamics (16).

$$WHL.K = DELAY3(RRR.JK,DEL)$$

WHL = Sales requests of WHoLesale (goods/day)

DEL = DELay in submitting sales requests to wholesale (days)

The purchase sales request rate then is adjusted to compensate for the difference between the actual level of inventory at retail and the manager's desired goal for maintain some level of inventory at retail. The system does not respond immediately to the error in inventory level, and the model must include a compensating adjustment time.

$$PSR.KL = WHL.JK + (1/SAT)(IDR.K-IAR.K)$$

PSR = Purchase Sales Requests (goods/day)

DEL = Delay in submitting sales request to wholesale (days)

SAT = Secondary Adjustment Time (days)

IDR = Inventory level Desired at Retail (goods)

The desired level of inventory is purely a managerial decision and is based upon the level of inventory he deems necessary to sustain operation over some fixed period of time plus a factor compensating for the amount of loss he may expect due to perishability. As in the case of the error between actual and desired inventory level, there is an adjustment time which must be included to provide for this perishability loss.

$$IDR.K = (AIR)(RSR.K) + (AIL)(ERR2.K)$$

AIR = proportionality constAnt between Inventory and average sales at Retail (days)

AIL = proportionality constAnt between Inventory and average perishavility Loss at retail (days)

RSR = actual sales Requests Smoothed at Retail (goods/day)

ERR2 = ERRor in inventory level given by daily perishability (goods/day)

The smoothed value of the actual sales requests is actually an averaged value computed over whatever period is required to accumulate appropriate information and subject to direct managerial control. The same is true of the smoothed value of the perishability rate, referred to as the error.

$$\text{RSR.K} = \text{RSR.J} + (\text{DT})(1/\text{DRR})(\text{RRR.JK}-\text{RSR.J})$$

DRR = requisition smoothing time constant (days)

$$\text{ERR2.K} = \text{ERR2.J} + (\text{DT})(1/\text{PAT})(\text{PER.JK}-\text{ERR2.J})$$

PAT = Primary Adjustment Time (days)

PER = PERishability rate (goods/day)

The rate at which goods are received at retail from wholesale is considered to be a function of the rate at which purchase sales requests are sent to wholesale. The time lag associated with having goods in transit is again provided for by a third order delay approximation between purchase sales requests rate and receipt of goods rate.

$$\text{SRR.KL} = \text{DELAY3}(\text{PSR.JK},\text{DTR})$$

DTR = Delay period in TRansit (days)

The perishability portion of the model may be thought of as a monotomically nondecreasing accumulation of perished goods.

$$LOS.K = LOS.J + (DT)(PER.JK)$$

LOS = LOSs due to perishability (goods)

The current level of accumulation is the sum of yesterday's level and that amount which has perished since. The perishability rate is a fraction of the current level of inventory.

$$PER.KL = (.01)(FRA.K)(IAR.K)$$

FRA = perishability function defined earlier (per cent/day)

As previously discussed the portion of inventory that perishes daily is a function of the length of time goods are delayed in inventory. The average delay may be determined by dividing the current inventory level by the sum of the average rates of flow of goods out of retail.

$$AVDL.K = (IAR.K)/(ATM.K + ATL.K)$$

AVDL = AVerage DeLay at retail (days)

ATM = Average rate of flow To Market (goods/day)

ATL = Average rate of flow To Loss accumulation (goods/day)

And the average rates of flow of goods out of retail both to market and to the accumulation of perished goods are calculated in a manner similar to the smoothed sales rate. The period of time over which the average is evaluated is subject to the manager's direct control and its variance might have some significance in his system's output.

ATM.K = ATM.J + (DT)(1/AVT)(SSR.JK-ATM.J)

ATL.K = ATL.J + (DT)(1/AVT)(PER.JK-ATL.J)

AVT = AVeraging Time for calculation (days)

As previously mentioned the actual sales rate of goods received at retail is considered to be a function of the potential sales rate, an independent input variable.

RRR.KL = DELAY3(AUX.JK, DTC)

AUX.JK = (.01)(FRAC.K)(PRR.JK)

PRR = Potential sales Rate at Retail (goods/day)

DTC = Delay in Telling Customers of inventory level (days)

AUX = AUXiliary variable

FRAC = probability sales rate function defined earlier (prob)

The actual sales rate is subjected to a delay related to customers becoming aware of what inventory is available at retail. Again the manager may exercise considerable influence over the length of this delay, advancing or retarding it to the degree he deems appropriate. A principal tool used by the manager in formulating such decisions is his lost sales rate. A measure of the difference between potential and actual sales, the lost sales rate may be evaluated as follows:

$$PLS.K = (100)(LRP.K-LRA.K)/LRP.k$$

PSL = Per cent Lost Sales (per cent)

LRP = Long Range average Potential sales rate (goods/day)

LRA = Long Range average Actual sales rate (goods/day)

The long range average potential and actual sales rates are merely smoothed versions of their current values, with the averaging constant determined at the manager's discretion.

$$LRP.K = LRP.J + (DT)(1/AVG)(PRR.JK-LRP.J)$$

$$LRA.K = LRA.J + (DT)(1/AVG)(RRR.JK-LRA.J)$$

AVG = averaging time constant (days)

The daily total cost rate for perishability and lost sales is the sum of the two individual daily cost rates.

$$TC.K = LSC.K + LGC.K$$

TC = Total Cost (dollars/day)

LSC = Lost Sales Cost (dollars/day)

LGC = Lost Goods Cost (dollars/day)

And the cumulative costs are as follows:

$$CTC.K = CTC.J + (DT)(TC.K/DAY)$$

$$\mathrm{CLSC.K = CLSC.J + (DT)(LSC.K/DAY)}$$

$$\mathrm{CLGC.K = CLGC.J + (DT)(LGC.K/DAY)}$$

CTC = Cumulative Total Cost (dollars)

CLSC = Cumulative Lost Sales Cost (dollars)

CLGC = Cumulative Lost Goods Cost (dollars)

DAY   time constant (days)

These equations together with the initial conditions under which the model was simulated represent the perishable, seasonal inventory system; their interdependence is illustrated in Figure 7 , and they are collected as a complete simulation model in Appendix B.

CHAPTER VI

MODEL ANALYSIS

Initial Conditions

Since the system being modeled presumedly exists in the time domain, certain initial conditions must be defined for the mathematical model to simulate its previous existance. One assumption normally made is that the system is at steady state with its environment. Hence, no error exists between desired and actual inventory level, and IAR = IDR. Further the smoothed sales rate equals actual sales rate, RSR = RRR, and the number of orders unfilled is merely a delayed value of the sales rate, UOR = (DFR)(RRR). The simulation is to be conducted over some period and only the accumulation of perished goods during that period is of interest. Therefore initially the accumulation is assumed to be a relative zero, LOS = 0, as is the fractional part of goods in inventory having perished, FRA = 0. The average rate that goods leave retail is initially equated both to perishability rate and shipment rate appropriately, ATM = SSR and ATL = PER. Similarily the long range actual and potential sales rates are equated initially to their respective current actual and potential sales rate, LRA = RRR and LRP = PRR. Finally it is assumed initially that there are no lost sales. Hence the probability that a potential sale is consumated is one, FRAC = 100.

The independent variable system input, potential sales in this case, is determined by the model designer to best approximate the condition being modeled. In this model to simulate seasonal fluctuations a sinesoidal input function was selected with a period of forty-five days. Additionally the function was designed to fluctuate about some basic constant number that a stable system might expect. The equations for this input function are as follows:

$$PRR.KL = RRI + RCR.K$$

$$RCR.K = (STH)SIN((2PI)(TIME.K)/45)$$

STH = amplitude

RRI = constant Rate of sales at Retail Initially (goods/day)

RCR = Requisition Change at Retail (goods/day)

Constants

The following parameters were considered variables in previous discussion, but are referred to and treated as constants in the simulation model, since they remain fixed for the duration of any given simulation "run". However, they are indeed variable in the sense that the model designer may vary their values in subsequent simulation "runs" to evaluate the model's sensitivity to any one or several such parameters. The values listed below will be considered the basic set which determine the existing model output, and variations of this set will be explored when stimulating and analyzing different model output.

AVG = 30 days

AIR = 8 days

AIL = 2 days

DFR = 1 day

SAT = 4 days

PAT = 10 days

AVT = 5 days

DEL = 2 days

DTC = 1 day

DRR = 8 days

DTR = 2 days

RRI = 1000 goods/day

STH = 100 goods/day

Model Output

In analyzing the output of an industrial dynamics model the analyst is chiefly concerned with the behavior patterns of the system's variables. Output patterns are compared with hypothetical behavior patterns, and their similarity (difference) tends to support (negate) the validity of the model. Model validation in industrial dynamics methodology is itself not esthetically appealing to more analytically oriented model builders. There are no clearly defined statistical test procedures available with which to measure the model output data and assign some degree of validity based on an arbitrarily selected significance level. The philosophy is that there may exist little correlation between data generated by the simulation model of a social system and data of similar dimension collected in a real system, since the latter may be distorted by a wide variety of environmental influence either not included or not adequately represented in the simulation model. This does not negate the possibility of model validation, however. Industrial dynamics model validation is more a subjective opinion of the model builder than an analytical correlation. His belief that his model includes the most significant feedback loops which truely represent the system's internal causal interactions, and a favorable comparison of the model output with his hypothetical behavior patterns serve to validate the model. The most definitive test of the model's validity is the reaction of the real system upon implementation of recommended changes suggested by the simulated model. The model may be considered ultimately valid if the real system's behavior patterns compare favorably with the model's contrived patterns.

Before discussing the output patterns of the basic model found in Appendix B, a brief discussion of the impact of sales rate and production rate on inventory level is of some value. As illustrated in Figure 14, when production increases the level of inventory increases, and as production peaks and drops back down inventory will continue to increase, but its function's slope will change from positive to negative. Finally production ceases and inventory stabilizes at the new level determined by the amount produced during the period in question. The sales rate has a similar but opposite effect on inventory level. The manager must be aware that even though he is no longer producing his inventory will not return to what he had previously established as an economically feasible level until it is acted upon by the sales rate.

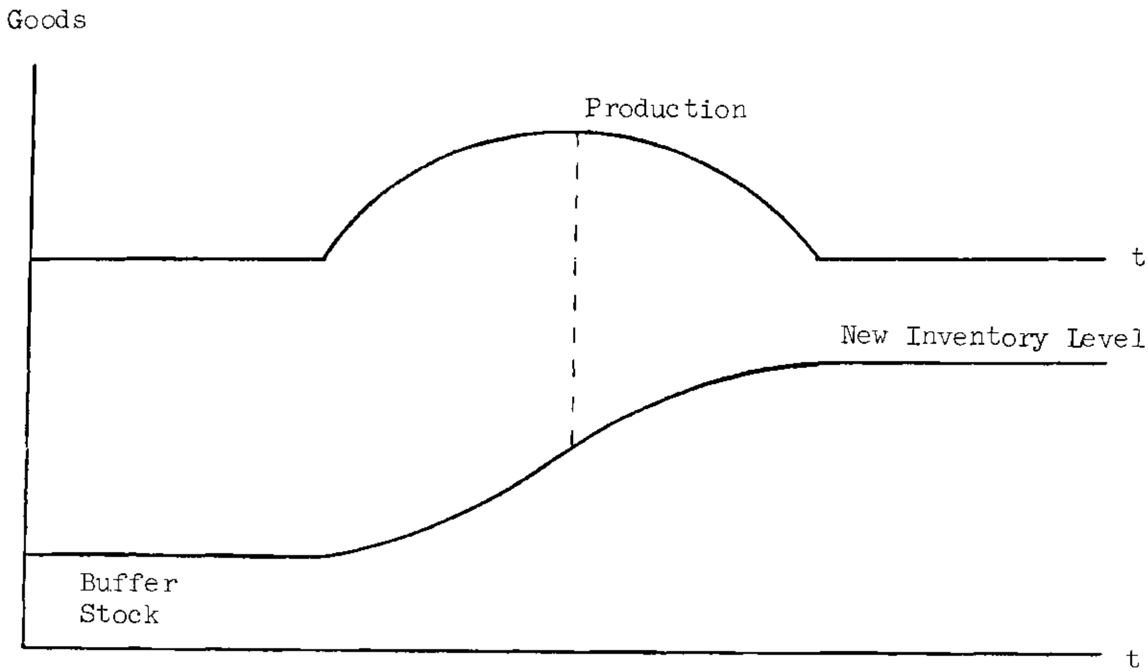


Figure 14. Inventory Growth Due to Production Variance.

The effect is illustrated in Figure 15. Note that maximum inventory occurs when decreasing production rate equals increasing sales rate. The output from the basic simulation model is illustrated in Figure 16. In the case of seasonally varying sales and production rates inventory level also varies, its period a measure of the time when production rate equals sales rate. Under the conditions of this initial run the model's production rate displays a greater amplitude than does sales rate. This might be explained as a tendency of the manager to over (under) order to meet rising (declining) demand. Further, production lags sales possibly due to the delay in responding to rising or declining sales rate manifest in hiring additional labor, setting up new machines, gathering necessary material, providing for additional transportation needs, etc. The results of this difference in patterns between production and sales rates is a varying inventory level which is maximum sometime after peak demand.

Intuitively inventory level, as a measure of the difference between what is produced and what is sold, will be minimized when production rate closely approximates sales rate. Further, in addition to minimizing perishability costs by minimizing inventory level, lost sales will also be minimized since the amount of underproduction or non-availability of goods for sale will be reduced by reducing the difference between production and sales rates. The ideal relationship between inventory level and sales rate, that being identical phasing, can exist only when production is exactly equal to or leading sales such that declining production equals sales when the latter is maximum. Clearly, this is impossible if production rate is dependent solely upon delayed

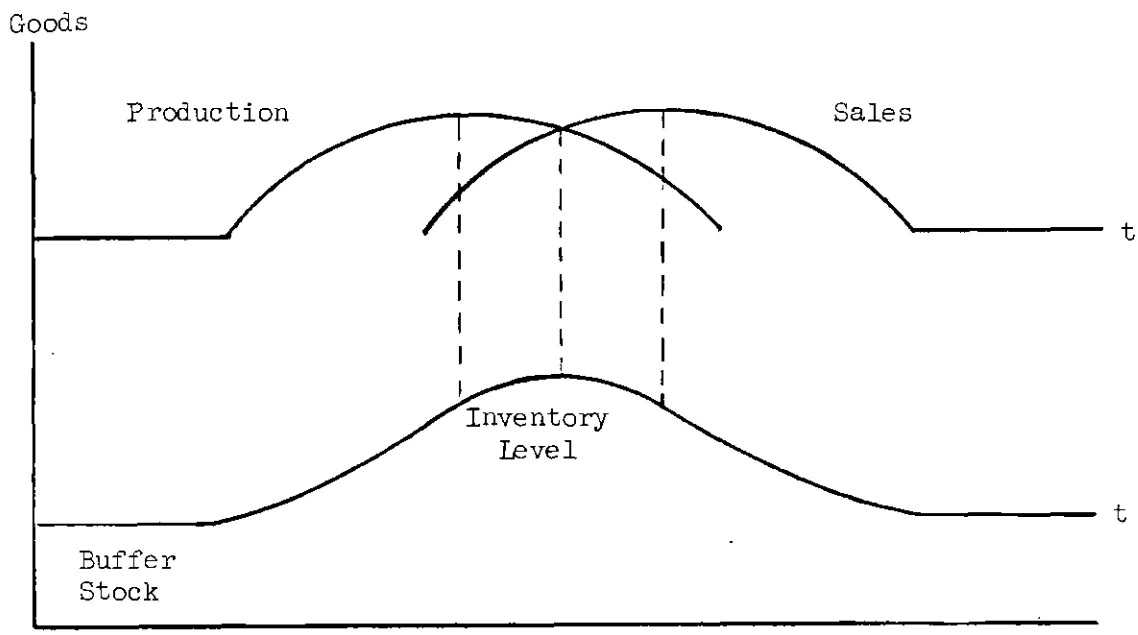


Figure 15. Inventory Level Adjustment Due to Production and Sales Variance.

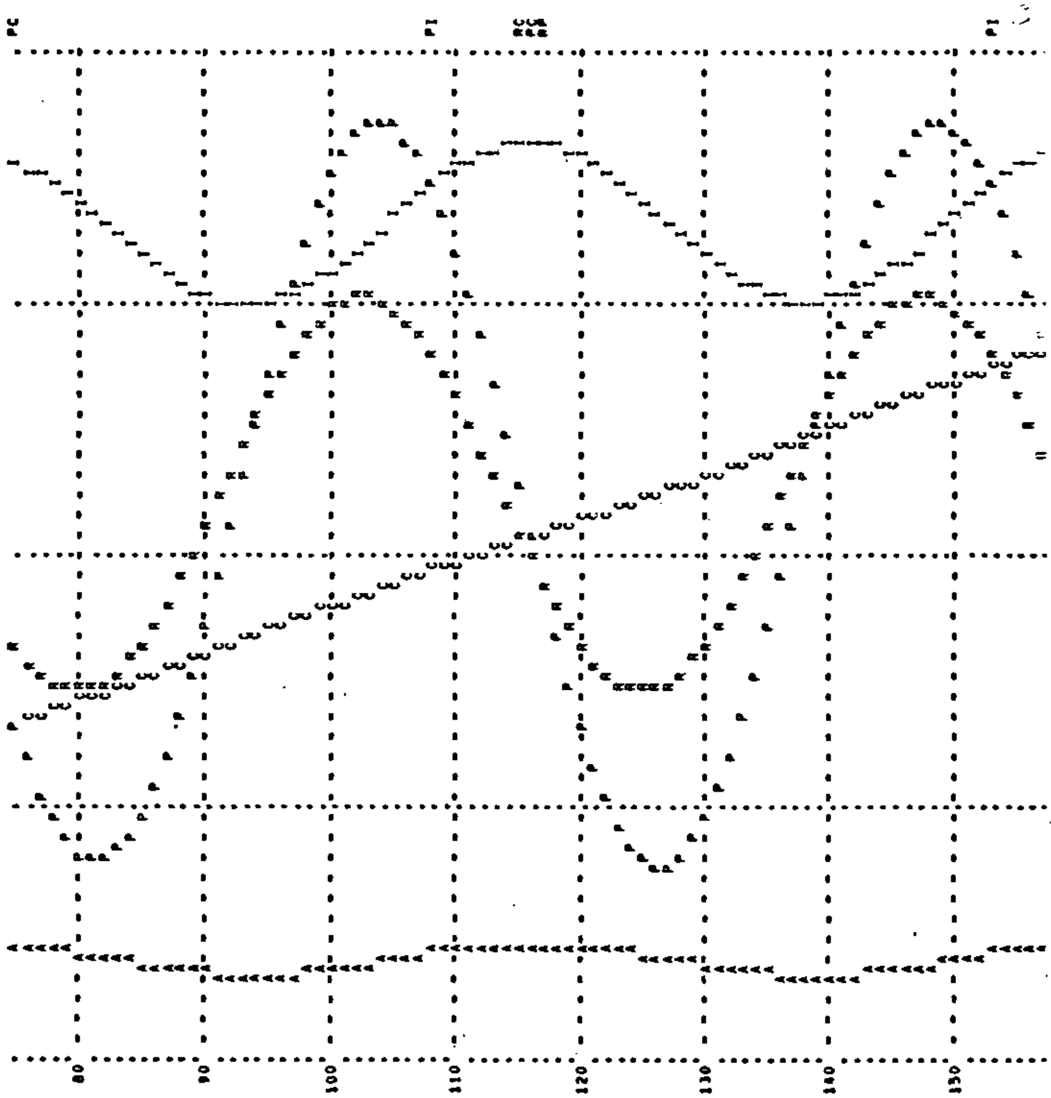


Figure 16. Extract of Appendix B, Basic Simulation Model.

information about the sales rate, and forecasting is inadequate. Nevertheless certain tools are available to the system manager in the form of inventory adjustment times and certain policies regarding the desired level of inventory with which he may within limits adjust the production rate more to his liking. Again in this model production rate is considered to be the rate at which goods are received at retail.

Before exploring the alternatives available to the manager for adjusting his inventory level certain related cost considerations are of interest. As was discussed in the previous chapter the costs involved with perishability and lost sales have an inverse functional relationship over time. A typical perishability cost function, illustrated in Figure 17, was constructed as an academic tool to portray the effect on cost of varying average delay. This function may be described in DYNAMO simulation language as follows:

LGC.K = TABHL (LG, AVDL.K,3.90,4.60,.05)

LG* = 4/5/6/7/9/11/14/17/20/24/29/34/39/44/52

As average delay varies the cost of perishability varies with a greater cost being assessed the longer goods are available at retail.

A similar lost sales cost function was constructed as illustrated in Figure 18. This function may be described in the DYNAMO simulation language as follows:

LSC.K = TABHL (LS,AVDL.K,3.90,4.60,.05)

LS* = 75/53/43/36/30/25/21/17/14/11/9/7/5/4/3

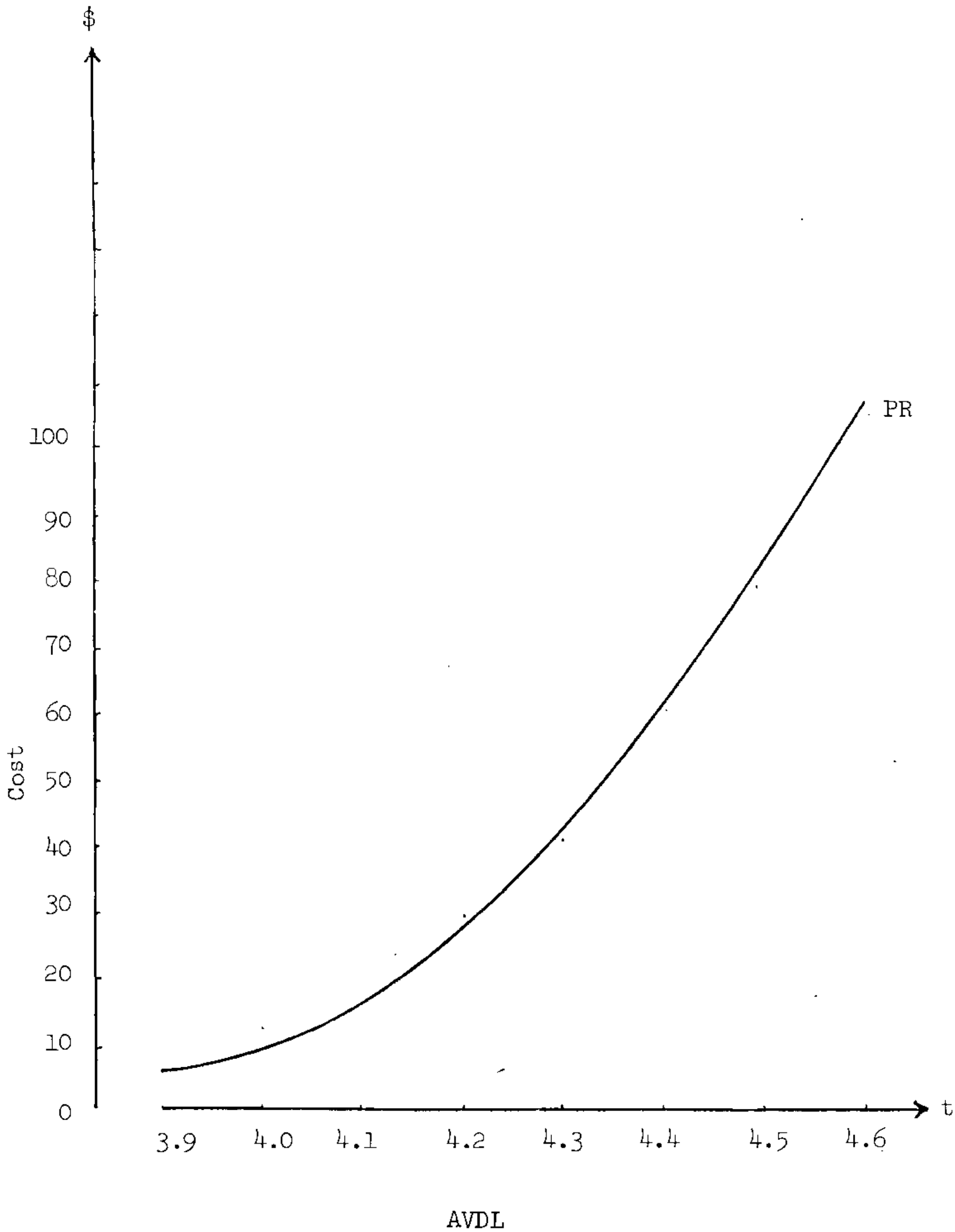


Figure 17. Model Perishability Cost Function.

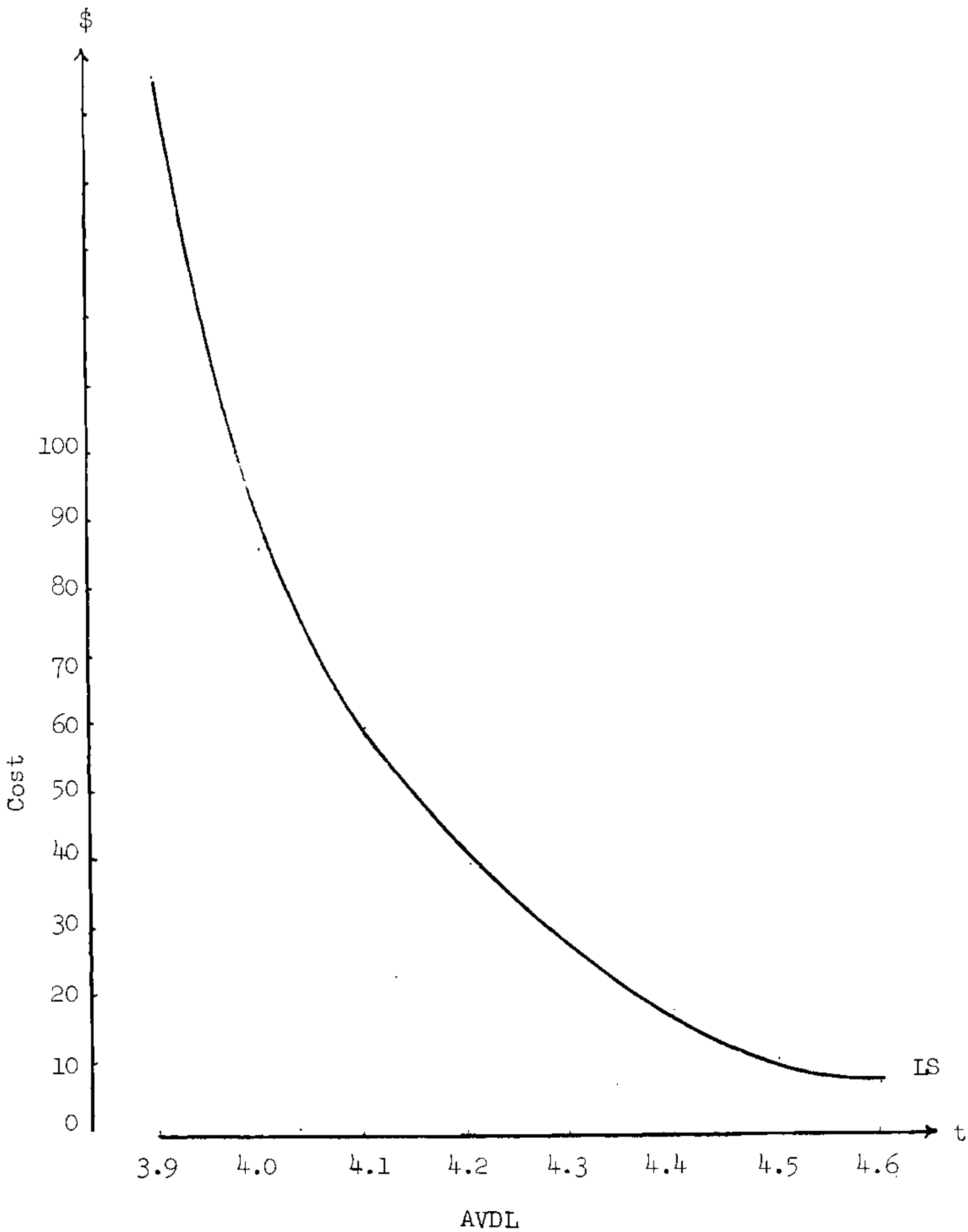


Figure 18. Model Lost Sales Cost Function.

The cost functions are combined to produce a total cost function in Figure 19. Notice that given the system conditions in the initial model run the manager would realize a daily perishability and lost sales cost of approximately six dollars and forty-three dollars respectively. Were the average delay extended from approximately four to four and one-half days the daily costs would then be only six dollars for lost sales but thirty-nine collars for perishability loss. Clearly, however, the minimum total cost of approximately thirty-four dollars daily occurs at an average delay of four and one-quarter days. Now that the manager recognizes the current status of his system (initial model run) and the more desirable situation, the problem with which he must deal lies in how to adjust his system to the preferred average delay, thereby minimizing perishability as well as lost sales as determined by the minimum total cost value.

Turning his attention back to the initial model run the manager sees little opportunity for bringing inventory level in phase with sales rate by attempting to adjust the latter, since in this system sales rate is defined to be merely a fraction of potential sales, the system independent variable input function. Since that fraction is a function of the average delay and exhibits little variation as the average delay is extended from four to four and one-half days (Figure 9), the manager can expect almost negligible phase shifting of the sales rate, so he turns his attention toward moving the production rate to more closely approximate the sales rate.

The production rate for this system is actually the rate at which goods become available at retail, defined to be a delayed value of

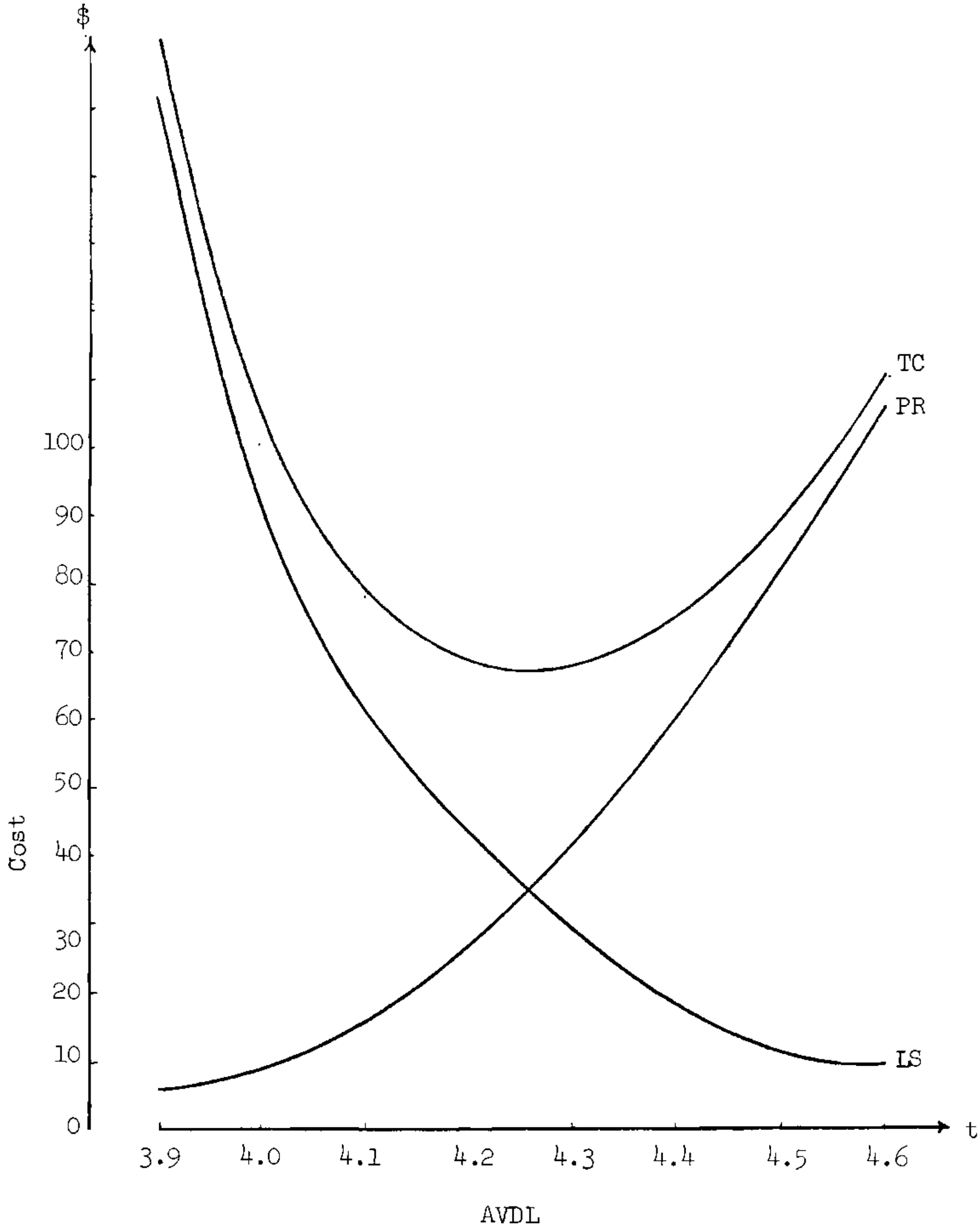


Figure 19. Model Total Cost Function.

the purchase sales requests which are sent to wholesale. The manager may attempt to reduce the amount that production lags sales by reducing the delay in transit from wholesale to retail as well as his own delay in sending orders to wholesale. This is not, however, his only alternative for adjusting production rate. For example, he notices that his purchase sales requests include not only orders to replace inventory depleted by sales but also orders to adjust his inventory imbalance reflected as the difference between his actual inventory level and that which he desires it to be. The desired inventory level is a function of smoothed values of sales rate and perishability rate. By extending the period over which these rates are smoothed the manager may stabilize his desired inventory level, thereby reducing the amplitude of his purchase sales request rate causing a similar effect on the production rate. Another adjustment time available to the manager for stabilizing his purchase sales request rate is the period he allows for correcting for the difference between desired and actual inventory level. By extending this adjustment time he may effect an even more stable purchase sales request rate. The results of adopting these changes are illustrated in Figure 20. As suspected the production rate reflects a reduced amplitude more closely approximating that of the sales rate. However, reducing the delays does not appear to have any appreciable effect on the phase angle difference between production and sales. An analysis of the effect of increasing the adjustment times without reducing the delays, that situation illustrated in Appendix C, shows that smoothing the production rate by itself actually increases the phase angle difference. Then by reducing the

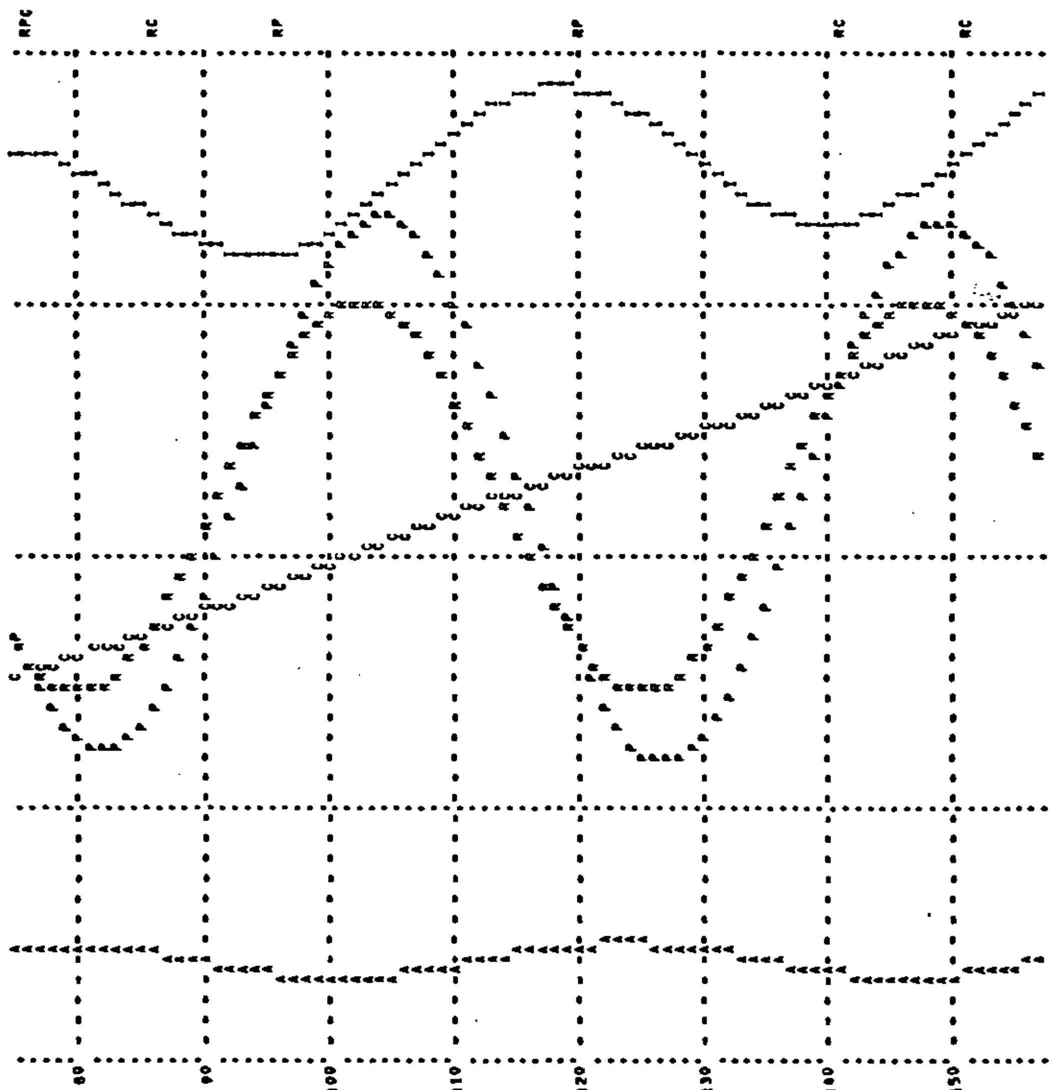


Figure 20. Extract of Appendix C, Effect of Smoothing Production Rate and Reducing Delays.

the phase angle is shifted back, allowing for a more stable production rate and inventory level without the increased cost associated with driving the inventory level farther out of phase with sales rate.

In spite of his more stable system the manager notices that he has accomplished no apparent improvement with respect to his total cost. A closer analysis indicates that by further stabilizing his system he has reduced the variance in the average delay. The effect was to shift the mean of the average delay slightly to the left (shorten the average delay), and reduce the standard deviation from the mean causing slightly higher daily lost sales costs and lower daily perishability costs and a generally more stable total cost rate.

The manager, although satisfied that increasing adjustment times and decreasing delays has added stability to his system, still desires to reduce total cost by increasing average delay. He decides that since his perishability costs are low while lost sales costs are high he should re-evaluate his policy concerning desired inventory level. Currently that policy provides for a proportionality constant between inventory and average sales of eight and between inventory and average perishability of two. By increasing either or both of these proportionality constants the desired inventory level will be raised forcing a related increase in production rate. The effect would be to expand the on-hand inventory level without a related increase in demand. Hence, more goods would be available longer to reduce lost sales at the expense of an increased perishability loss rate. Figure 21 illustrates the effect of increasing the proportionality constant between inventory and perishability from two to three. Notice that the

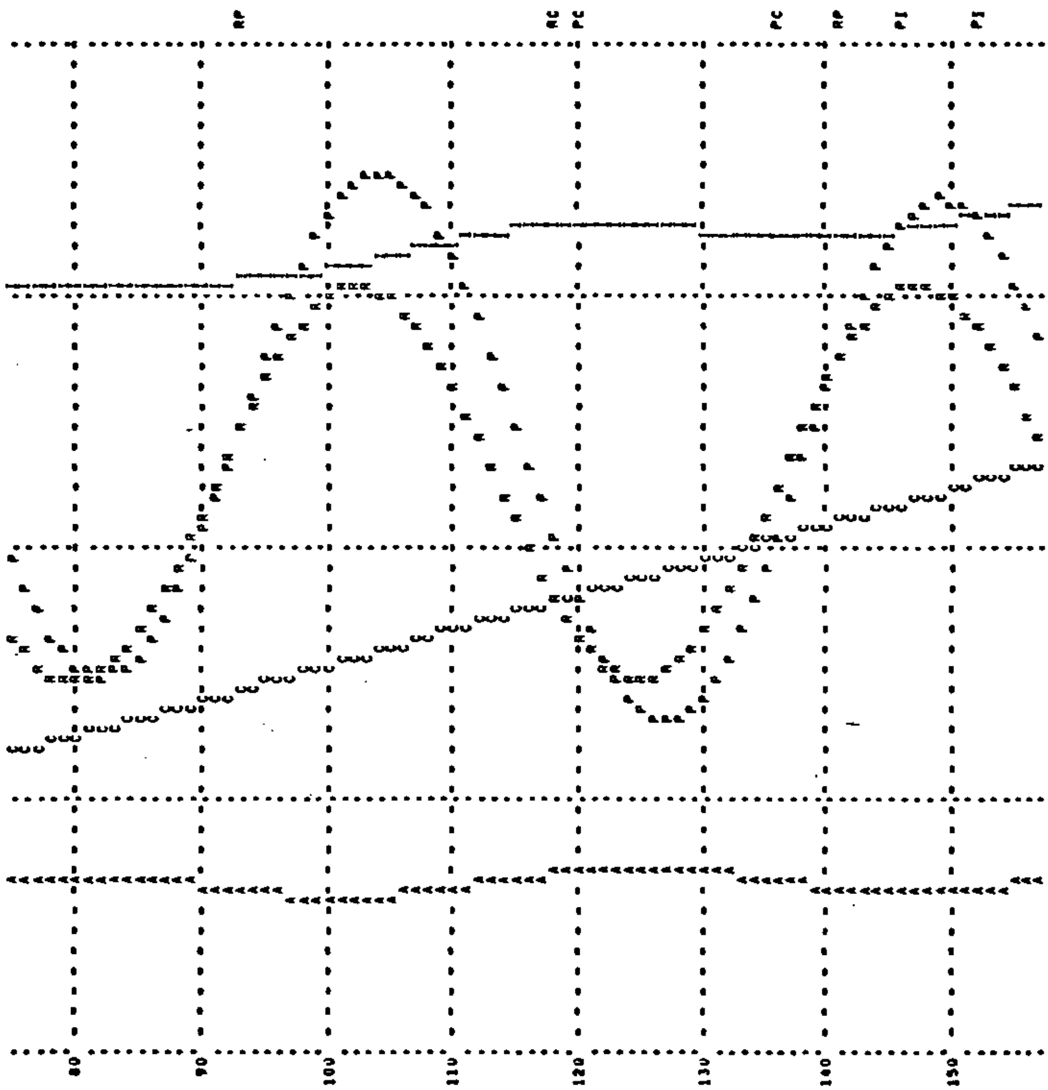


Figure 21. Extract of Appendix C, Effect of Smoothing Production Rate, Reducing Delays, and Increasing AIL.

average delay has been increased to approximately four and one-quarter days. The results are reduced daily lost sales cost by approximately 60 per cent, and an overall reduction of total cost by approximately 20 per cent. The impact of raising the proportionality constant without smoothing the production rate and decreasing its delay is illustrated in Appendix C. Although the comparison to that between the initial model and its more stable version, one additional significant consideration should be pointed out. Stabilizing the inventory is now even more important since a greater amount of capital is invested in on-hand inventory. With reduced inventory variation that capital is available to take advantage of other opportunities which would otherwise be lost.

Finally some consideration should be given to artifically adjusting the average delay by means of refrigeration, improved handling techniques, advertising, hiring additional personnel, etc. When analyzing the desirability of introducing any or all of these innovations the manager must first establish his new perishability and/or lost sales function as defined by this potential change. Next he must adjust his cost curves to reflect the additional cost associated with adding the techniques under analysis as well as the adjusted slope relationship that the techniques will produce. Then, by analyzing the new system output in a manner similar to the analysis herein he may be better prepared to decide on the appropriateness of undertaking the innovation under advisement.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The previous chapter showed the behavior of this model both initially and under some selected changes. The original model's output patterns respond quite similarly to the expected behavior with respect to the relationship between inventory level, production rate, and sales rate. As alternatives were introduced behavior patterns were subsequently adjusted reflecting a varying degree of sensitivity based on the nature and severity of the alternative under consideration. The system manager is provided with the following conclusions regarding control of perishable, seasonal inventory.

(1) The most significant variable in this system with which the manager may influence control is the average amount of time goods are available at retail. By an in-depth analysis of the variables and parameters within his system which contribute to this average delay and the factors which cause it to shift, the manager may establish a means of control necessary for efficient management of this type system.

(2) Properly defined cost functions for the product or class of products to be controlled are most useful and meaningful for controlling inventory. Because of their significance in defining the most desirable range over which the average delay may vary, considerable effort should be directed toward quantifying the costs associated with perishability

and lost sales.

(3) Given the appropriate variable functions this model provides the manager a means of determining the degree of responsiveness necessary both within his own sector to processing orders to wholesale and within the wholesale to shipping goods to retail.

(4) This model also provides the manager a better means of deciding how much inventory he should have on-hand to compensate for perishability without sacrificing sales potential. As was illustrated in the previous chapter an improper balance between goods to sales or goods to perishability can result in a significant increase in total cost. Upon validation and implementation of this model that balance may be more clearly defined with a possible subsequent improved cost situation.

Finally the technique of analysis used in this research, Industrial Dynamics, is indeed appropriate for identifying the significant internal causal relationships of a perishable, season inventory control system. No attempt was made to establish any precise values for "optimizing" inventory level for any particular product. Yet, by identifying the significant feedback loops and the relationships between then with only general, typical variable definitions a model was constructed through which considerable control may be exercised over this system. Now, given the feedback loops which are considered most important in this model, validation by implementation may be tested by setting up a small system of actual goods and related data and analyzing its interaction.

Recommendations

As a result of this research the following recommendations are suggested:

(1) A companion study should be conducted analyzing the environmentally broader problem associated with the production and market sectors.

(2) This model should be incorporated into the broader model and tested to identify any significant variance from behavior patterns already contrived.

(3) Because of the effect of changing the hypothetical perishability and sales probability functions, factual data representing these variables should be collected in order to achieve some measure of validity with a real system.

(4) This model should be restructured to include actual cash flow feedback loops in order to adequately evaluate the tradeoff balance between lost sales and perishability loss.

APPENDIX A

TYPICAL SYSTEMS

In an effort to provide a realistic foundation and some measure of validity for the theoretical model discussed in this thesis interviews were conducted with managers of retail perishable, seasonal inventory control enterprises in the vicinity of Atlanta, Georgia. The general nature of business conducted by these merchants and supporting operational policies are discussed below.

(1) Frank A. Smith Nursery
4020 Roswell Road, N. W.
Atlanta, Georgia

General

Mr. Smith's nursery has long been established in the Atlanta area, serving the needs of the public for approximately twenty-five years. His inventory includes an extremely wide range of products, the majority of which is perishable. He has a unique policy of no commercial advertising, relying on superior quality rather than quantity to enlist future demands.

Although Mr. Smith's business includes considerable local production of perishable goods, this by no means satisfies his overall operational needs. Annually he must travel extensively, analyzing the wholesale market in order to evaluate the best means of obtaining adequate amounts of quality products, which must be timely accessable as well as economical. Although his principal market is landscaping, he must also satisfy demands from an extremely broad market for perishable as well as related durable goods.

Requisition Processing

There is essentially no delay in processing demands. Goods are shipped into retail daily in small quantities to satisfy anticipated demands while minimizing loss due to handling and perishability. The volume shipped to retail varies depending on forecasted demands. This forecasting is based upon Mr. Smith's total experience rather than periodic data, knowledge of extreme weather conditions during the in-season product's growing period, and intuitive estimate of changing market taste or style.

Purchase Receipt Process

Although as mentioned some production is local there remains varying amounts of delay in shipping products to retail. Delay due to weather conditions (production retarded by extreme temperatures) is for the most part anticipated and provided for my making substitute products available. Delays due to other causes such as transportation and distance pose a significant problem. As will be discussed in more detail later the policy of emphasizing quality merchandise has the effect of smoothing if not altogether eliminating this problem.

On-hand Inventory

On-hand inventory is stored in a controlled environment greenhouse. Shelf life may be extended almost indefinitely, although capacity, storage costs, and seasonal constraints limit the feasibility of such an operating policy. Additionally products marketed mainly on their esthetic value are greatly dependent on the appearance of their blossoms. Hence the withered petals of a potted chrysanthemum, for example, would be tantamount to a perishability loss even though the plant itself had not perished.

Seasonality

Market demands for seasonal products are anticipated and a combination of working overtime and hiring additional labor is employed to meet the additional workload. Backorders are accepted, and in the case where insufficient inventory is available to satisfy immediate demands, those demands are lost sales and satisfying backorders takes priority. The decision as to how much production capacity is allocated to satisfying backorders and how much to supplying on-hand inventory is reached by closely monitoring market and weather conditions. It is noteworthy that this is most critical, since demands drop off sharply immediately after the season.

Lost Sales

Lost sales are of vital concern in this enterprise. To avoid them various policies are in effect, most notably the emphasis on quality. Mr. Smith feels that because of the quality of his product and the wide range of products available, only 1 per cent of his market is actually lost. Additionally his market is stabilized because of the long period over which he has been satisfying customers. Because of the high degree of product quality customers normally agree, if necessary, to wait delivery of desired goods rather than become lost sales with related lost good will. Additionally the wide range of products offers the opportunity to satisfy the most unique demands as well as the flexibility of product substitution for demands on unavailable goods. Finally the stable market and good product quality serve as a source of advertisement without realizing the associated additional cost.

(2) Borg's Florist, Inc.
2293 Candler Road
Decatur, Georgia

General

Borg's Florist is an established business in Decatur, Georgia and has been serving the needs of that community for approximately thirty years. The general nature of the product dealt with is indeed perishable, and inventory is stored in a refrigerated room. One of the principal markets is funeral services, although other market demands such as birthdays, anniversaries, weddings and appropriate seasons require attention to varying degrees.

Requisition Processing

Requisitions are generally processed telephonically. The delay between receiving and filling a requisition varies somewhat with a mean of approximately one day. In order to minimize that delay needs are anticipated based on past operating experience. Goods are continuously being received from the producer based on a priori purchase sales. Seasonal items are handled in a similar manner with a priori purchases from the producer, anticipating demands by a matter of weeks rather than days. It is noted that in this particular case seasonal goods, though perishable, are more durable than daily stockage since the former normally consists of potted plants rather than cut flowers.

Purchase Receipt Process

Delay in receiving goods from the producer is almost negligible. Seasonality has a favorable if any effect on this condition since seasonal demands are also anticipated by the producer and goods are made more readily available. An operating policy which aids in accomplishing this delay minimization is continuously dealing with the same producer. This has the desirable effects of establishing system stability in this interaction, confidence in product availability both quantitatively and qualitatively, and production or ordering cost stability for the retailer.

On-hand Inventory

This florist's policy is to maintain goods on-hand for a fixed period per product and then dispose of remaining inventory as waste. By way of clarification the period is fixed by means of product quality, and the product's shelf life is not extended by means of price reduction when the quality falls below a certain standard. In the case of carnations the shelf life is approximately one week with approximately five-hundred received daily from the producer.

Seasonality

As mentioned seasonality is adjusted for by anticipating market demands. Forecasting these demands consists of analyzing the previous season's data. Additional workload resulting from increased demands is provided for by working overtime in the case of short seasons (Mother's Day, Easter). Longer seasons require both in depth prior preparation and extended overtime working periods. Additional temporary labor is rarely employed due to the nonavailability of skilled personnel.

Lost Sales

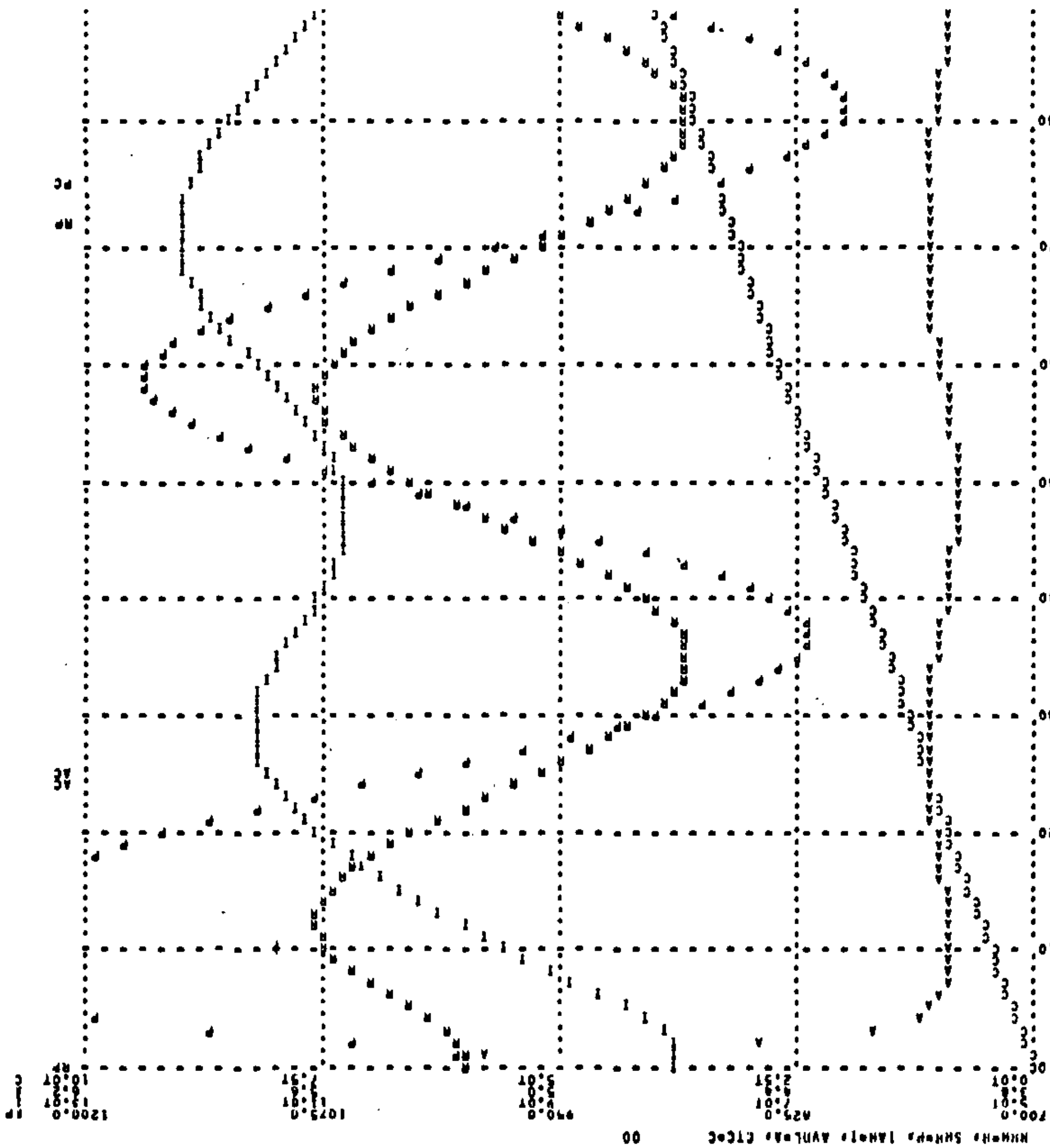
It appears that a principal factor in implementing operating policy is the desire to minimize lost sales. For this particular florist the problem is considered to be far more significant than perishability. Considerable effort is extended to avoid lost sales and the inherent ill will associated with a lost sale. Product substitutability is an effective means of reducing both lost sales and perishability. Upon nonavailability of a given product demanded it may be substituted for by a cheaper product possibly, at a subsequent profit loss of price difference but savings based upon avoiding a lost sale and the substituted product's perishability factor. In the absence of a suitable substitute this florist will assume the additional cost of obtaining the desired goods elsewhere in order to avoid lost sales.

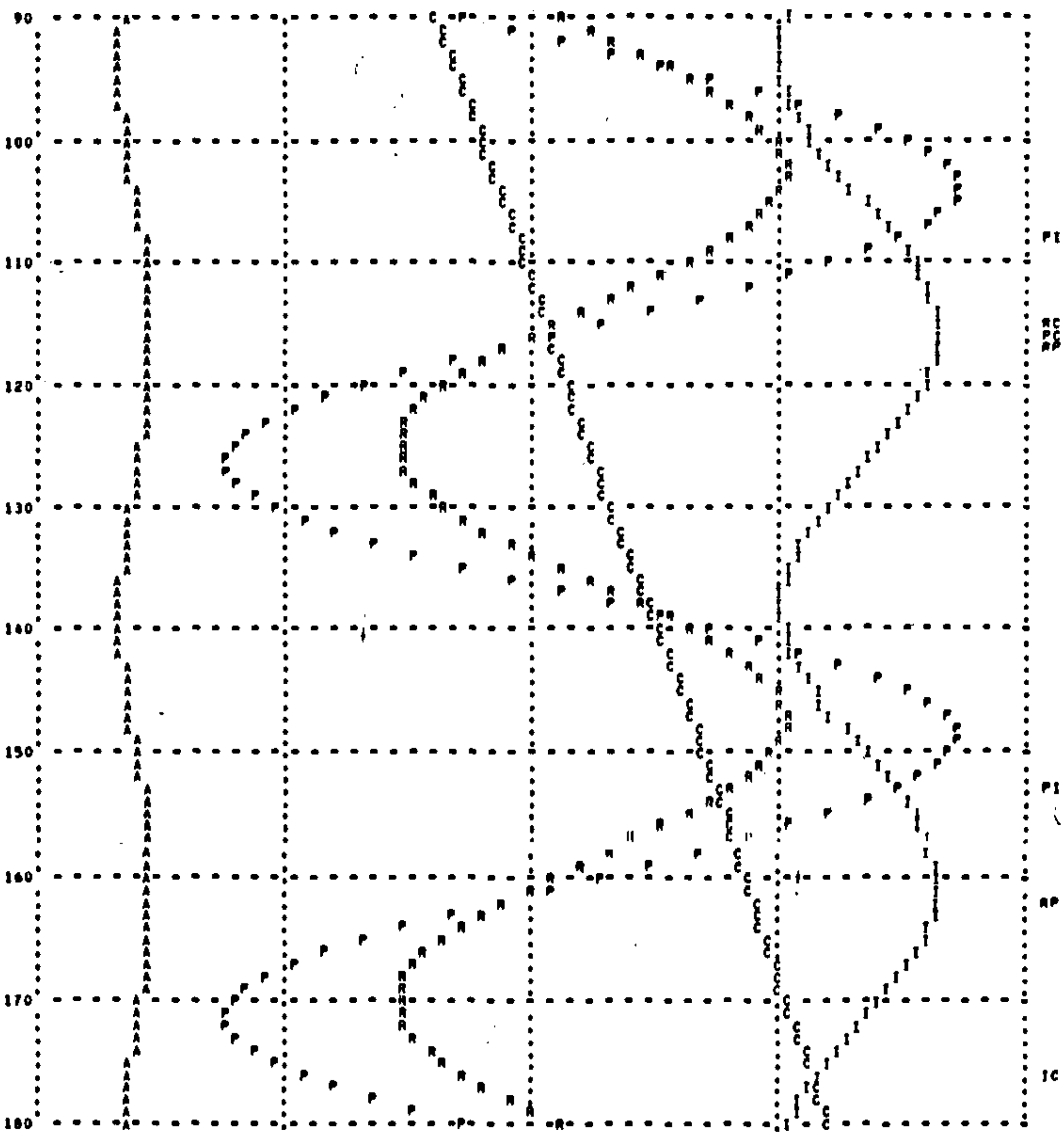
APPENDIX B

BASIC SIMULATION MODEL

```
MODEL OF RETAIL STORE, PERISHABLE GOODS

IAR.K=IAR.J+(DT)(SRR.JK-SSR.JK)
UOR.K=UOR.J+(DT)(RRR.JK-SSR.JK)
NIR.K=IAR.K/DT
STR.K=UOR.K/DFR
SSR.KL=MIN(STR.K,NIR.K)
WHL.KL=DELAY3(RRR.JK,DEL)
PSR.KL=WHL.JK+(1/SAT)(IDR.K-IAR.K)
IDR.K=(AIR)(RSR.K)+(AIL)(ERR2.K)
RSR.K=RSR.J+(DT)(1/DRR)(RRR.JK-RSR.J)
ERR2.K=ERR2.J+(DT)(1/PAT)(PER.JK-ERR2.J)
SRR.KL=DELAY3(PSR.JK,DTR)
LOS.K=LOS.J+(DT)(PER.JK+0)
PER.KL=(.01)(FRA.K)(IAR.K)
FRA.K=TABHL(DEC,AVDL.K,0,14,1)
DEC*=0/1/3/7/16/25/38/50/62/75/84/93/97/99/100
AVDL.K=(IAR.K)/(ATM.K+ATL.K)
ATM.K=ATM.J+(DT)(1/AVT)(SSR.JK-ATM.J)
ATL.K=ATL.J+(DT)(1/AVT)(PER.JK-ATL.J)
RRR.KL=DELAY3(AUX.JK,DTC)
AUX.KL=(.01)(FRAC.K)(PRR.JK)
FRAC.K=TABHL(SAL,AVDL.K,0,6,1)
SAL*=0/50/78/90/98/99/100
LRA.K=LRA.J+(DT)(1/AVG)(AUX.JK-LRA.J)
LRP.K=LRP.J+(DT)(1/AVG)(PRR.JK-LRP.J)
PLS.K=(100)(LRP.K-LRA.K)/(LRP.K)

COSTS

LSC.K=TABHL(LS,AVDL.K,3.90,4.60,.05)
LS*=75/53/43/36/30/25/21/17/14/11/9/7/5/4/3
LGC.K=TABHL(LG,AVDL.K,3.90,4.60,.05)
LG*=4/5/6/7/9/11/14/17/20/24/29/34/39/44/52
TC.K=LSC.K+LGC.K
CTC.K=CTC.J+(DT)(TC.K/DAY)
CLSC.K=CLSC.J+(DT)(LSC.K/DAY)
CLGC.K=CLGC.J+(DT)(LGC.K/DAY)

INITIAL CONDITIONS

UOR=(DFR)(RRR)
RSR=RRR
ERR2=0
IAR=IDR
LOS=0
FRAC=100
ATM=SSR
ATL=PER
FRA=0
LRA=AUX
LRP=PRR
TC=0
CTC=0
LSC=0
CLSC=0
LGC=0
CLGC=0
```

```
        INPUT

        PRR.KL=RRI+RCR.K
        RCR.K=(STH)SIN((2PI)(TIME.K)/45)

        CONSTANTS

        AVG=30
        AIR=8
        AIL=2
        DFR=1
        SAT=4
        PAT=10
        AVT=5
        DEL=2
        DTC=1
        DRR=8
        DTR=2
        DAY=1
        RRI=1000
        STH=100

PRINT   1)IAR,IDR/2)UDR/3)PRR,FRAC/4)RRR,SSR/5)PSR,SRR
PRINT   6)FRA,AVDL/7)ATM,ATL/8)PER,LOS/9)LRP,LPA/10)PLS
PRINT   11)FRR2/12)LSC,CLSC/13)LGC,CLGC/14)TC,CTC
PLOT    RRR=R,SRR=P(700,1200)/IAR=I/AVDL=A(3,8,6,2)/CTC=C(0,10000)
PLOT    LSC=S,LGC=G,TC=T(0,80)/CLSC=X,CLGC=Y(0,10000)
SPEC    DT=0.1/LENGTH=180/PRTPER=1/PLTPER=1
```

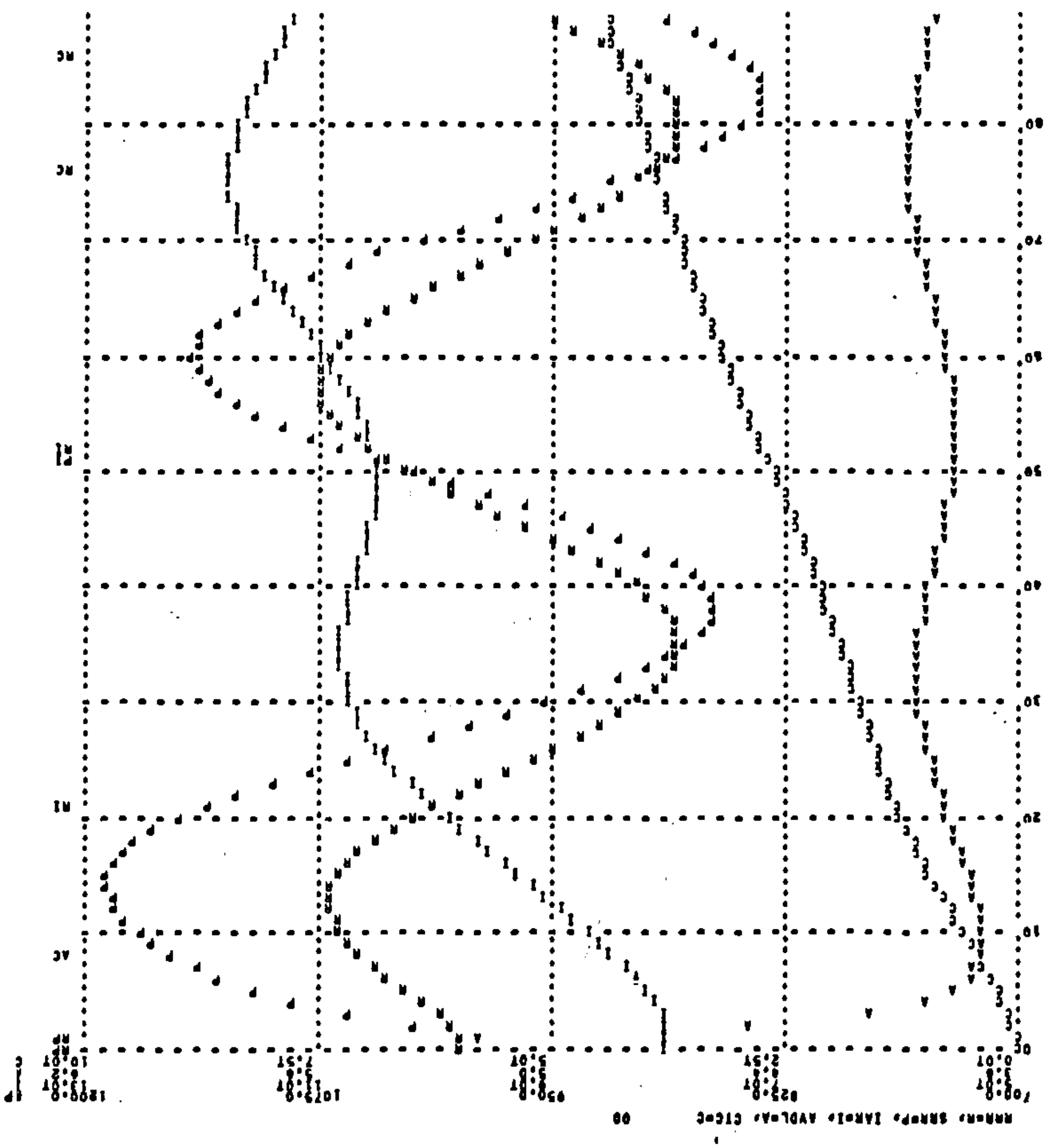


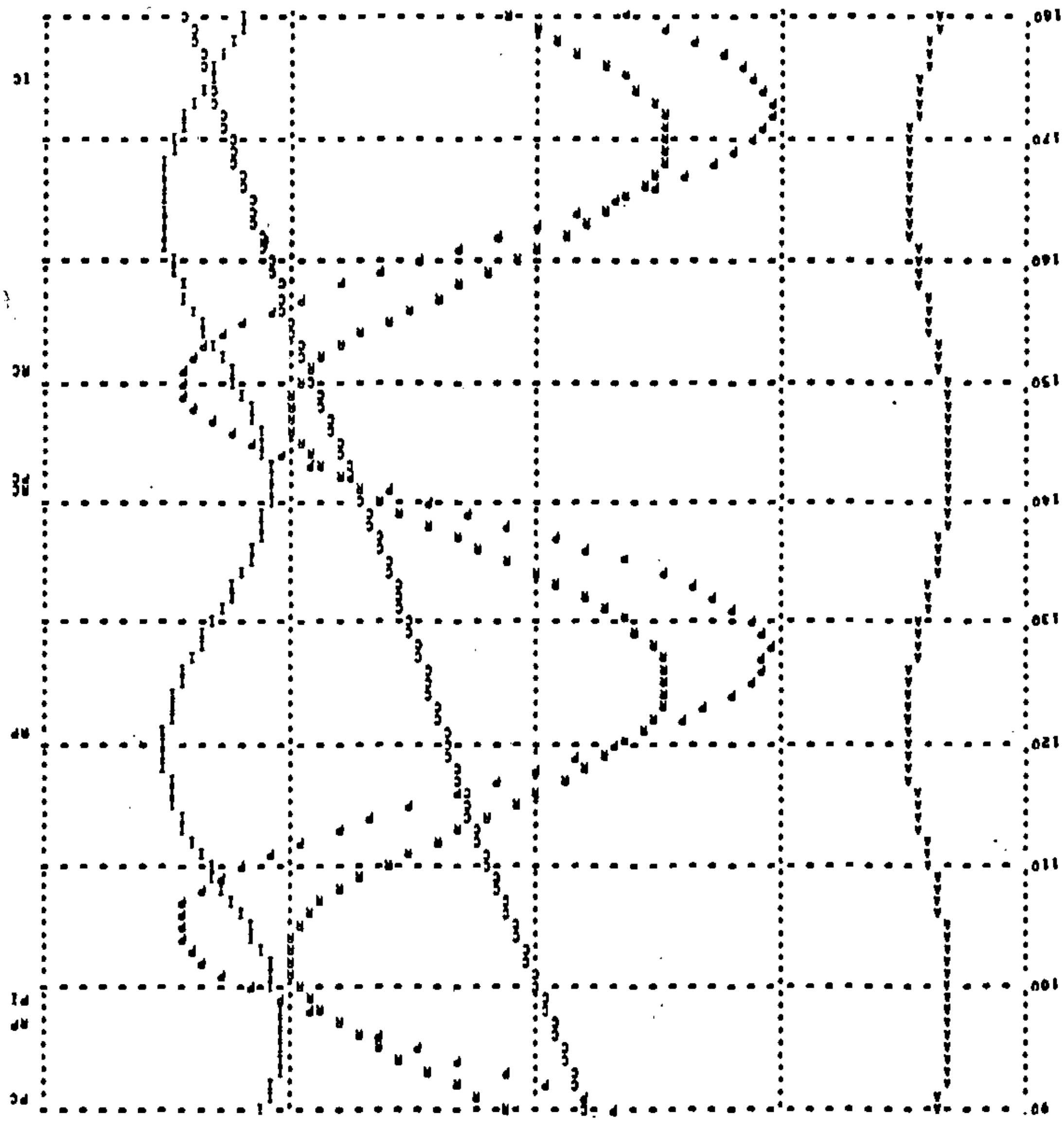

90
100
110
120
130
140
150
160
170
180
PI
RC
PC
RP
PI
RP
IC

APPENDIX C

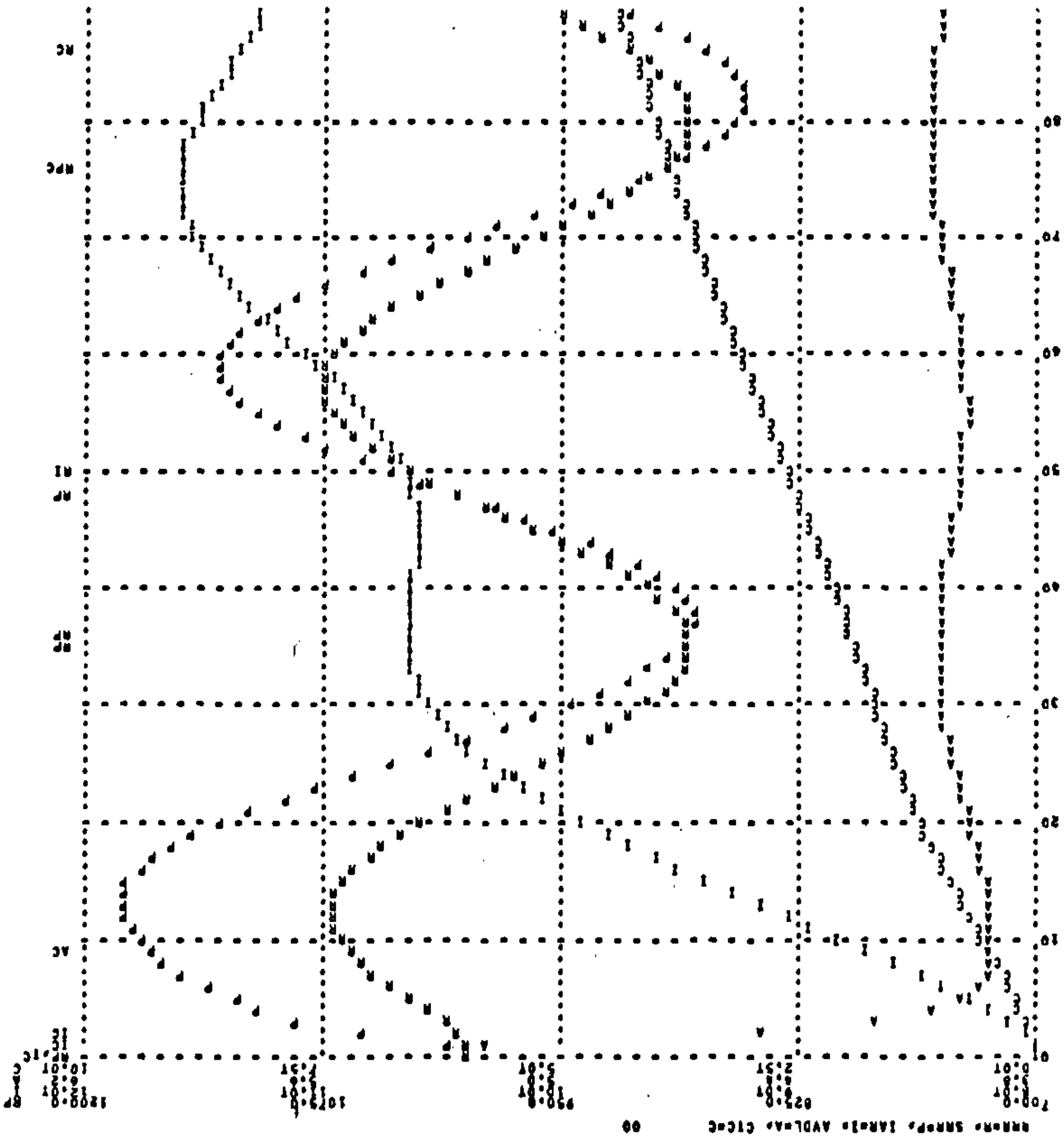
COMPARATIVE SYSTEM SIMULATIONS

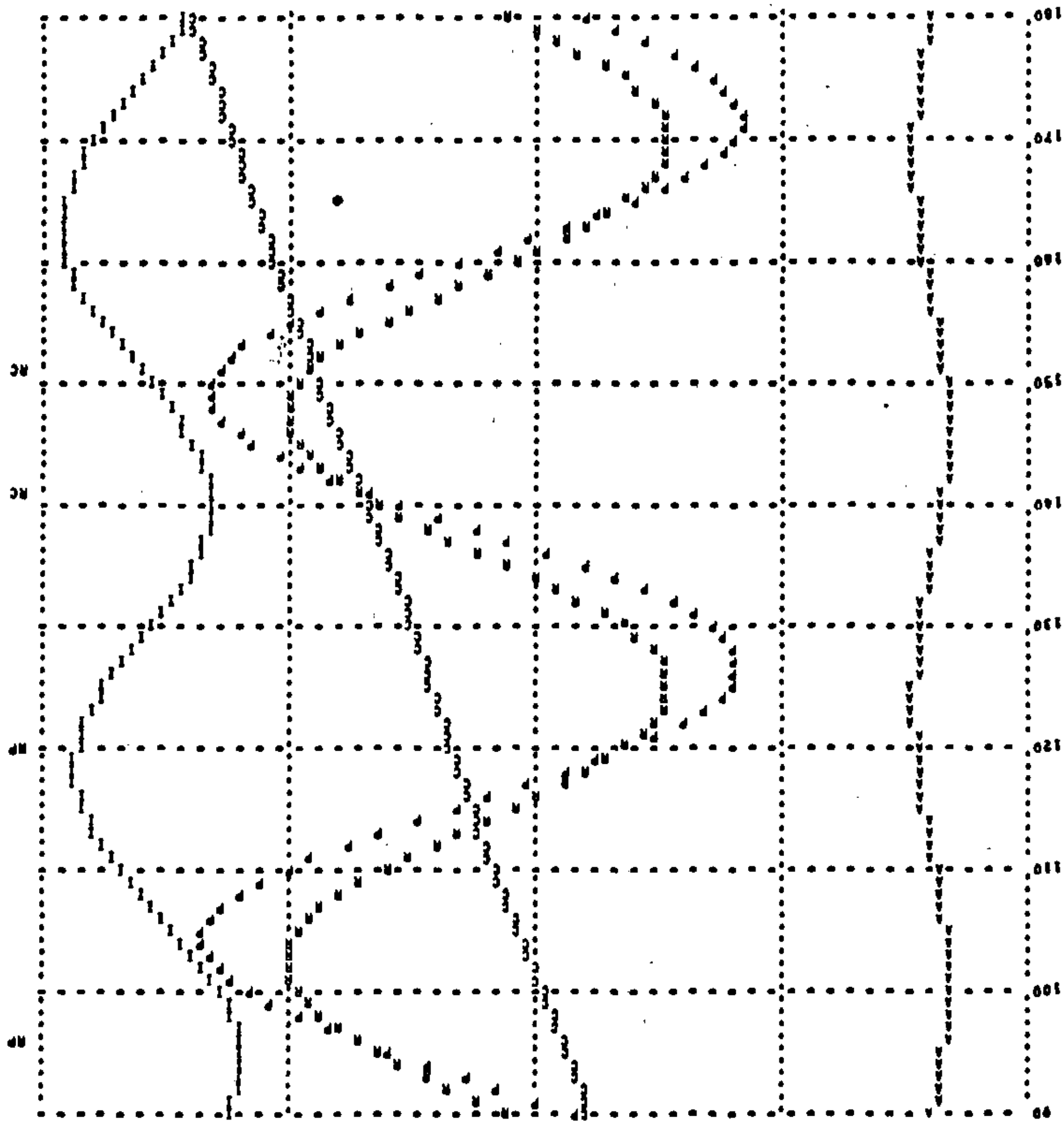
Basic Model with Smoothed Production Rate





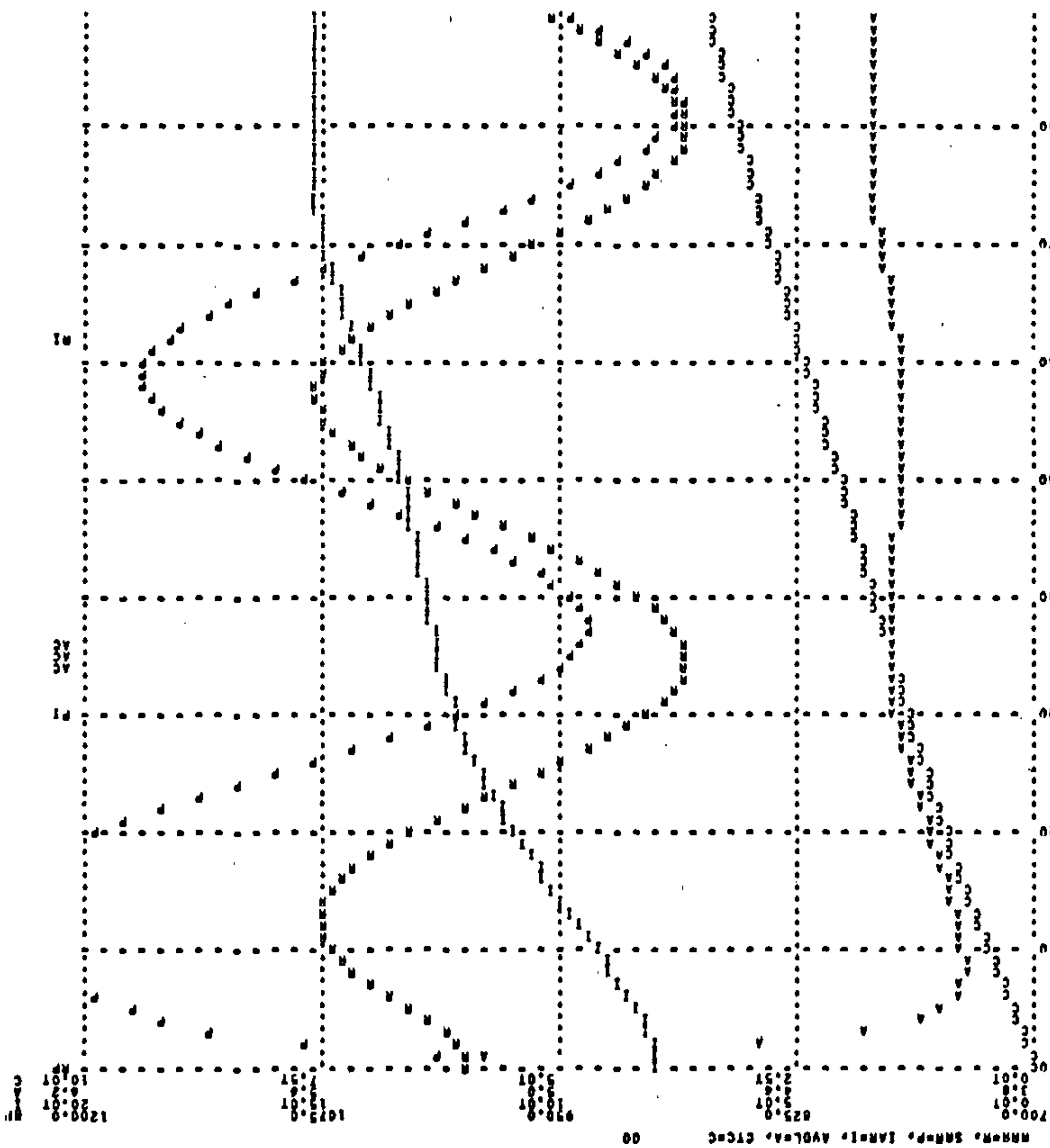
Basic Model with Smoothed Production Rate and Reduced Delays

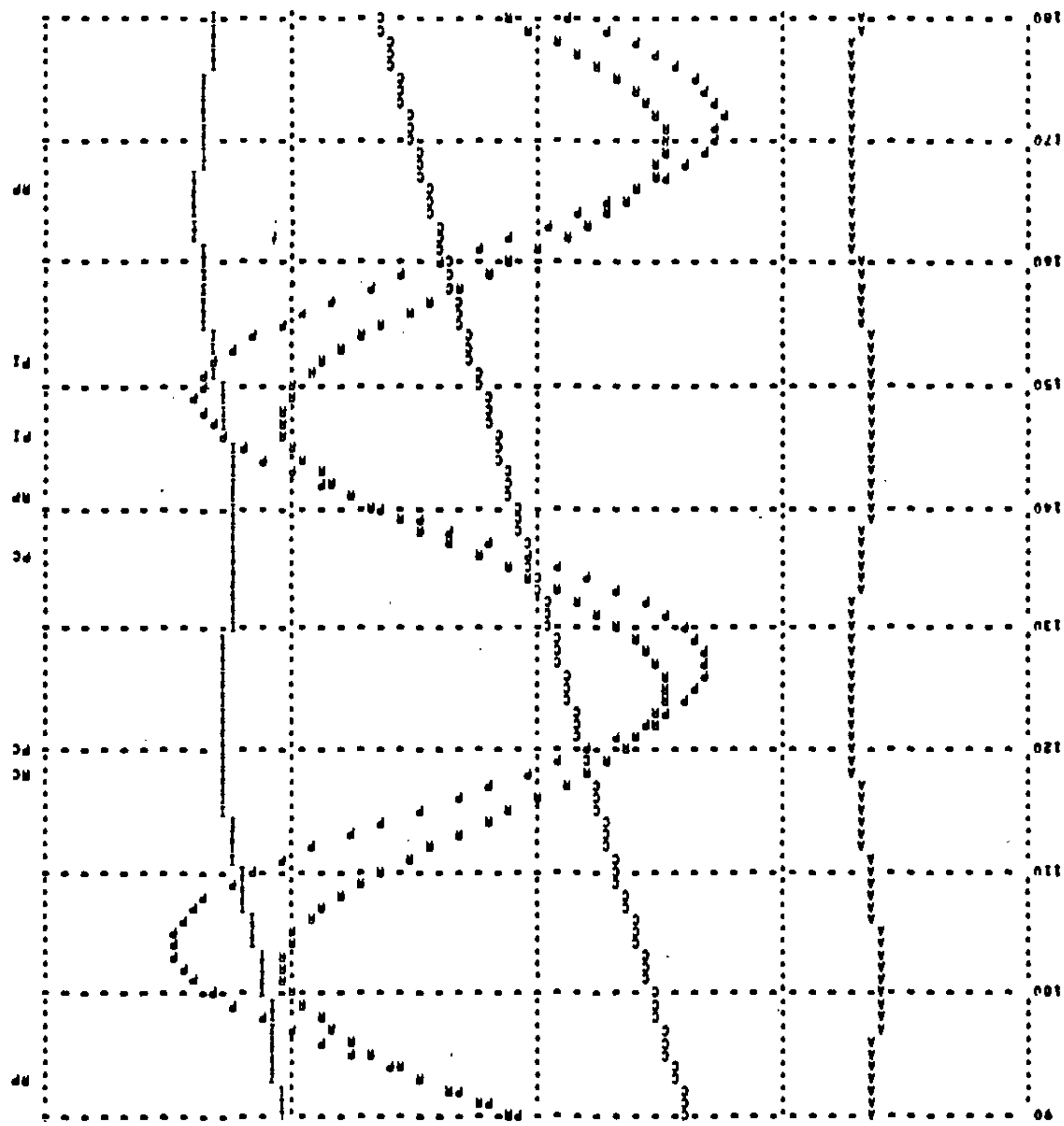




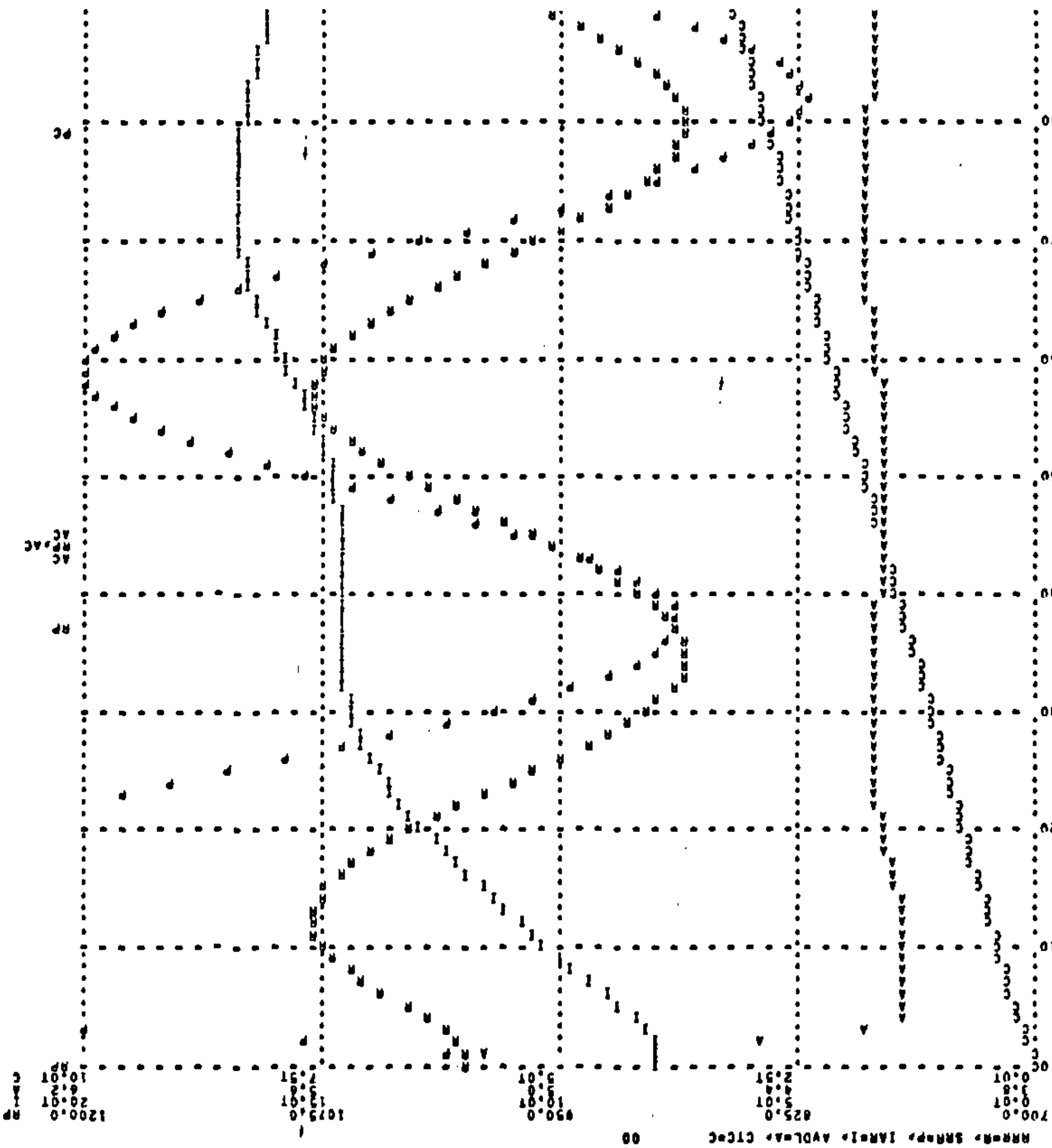
Basic Model with Smoothed Production Rate,

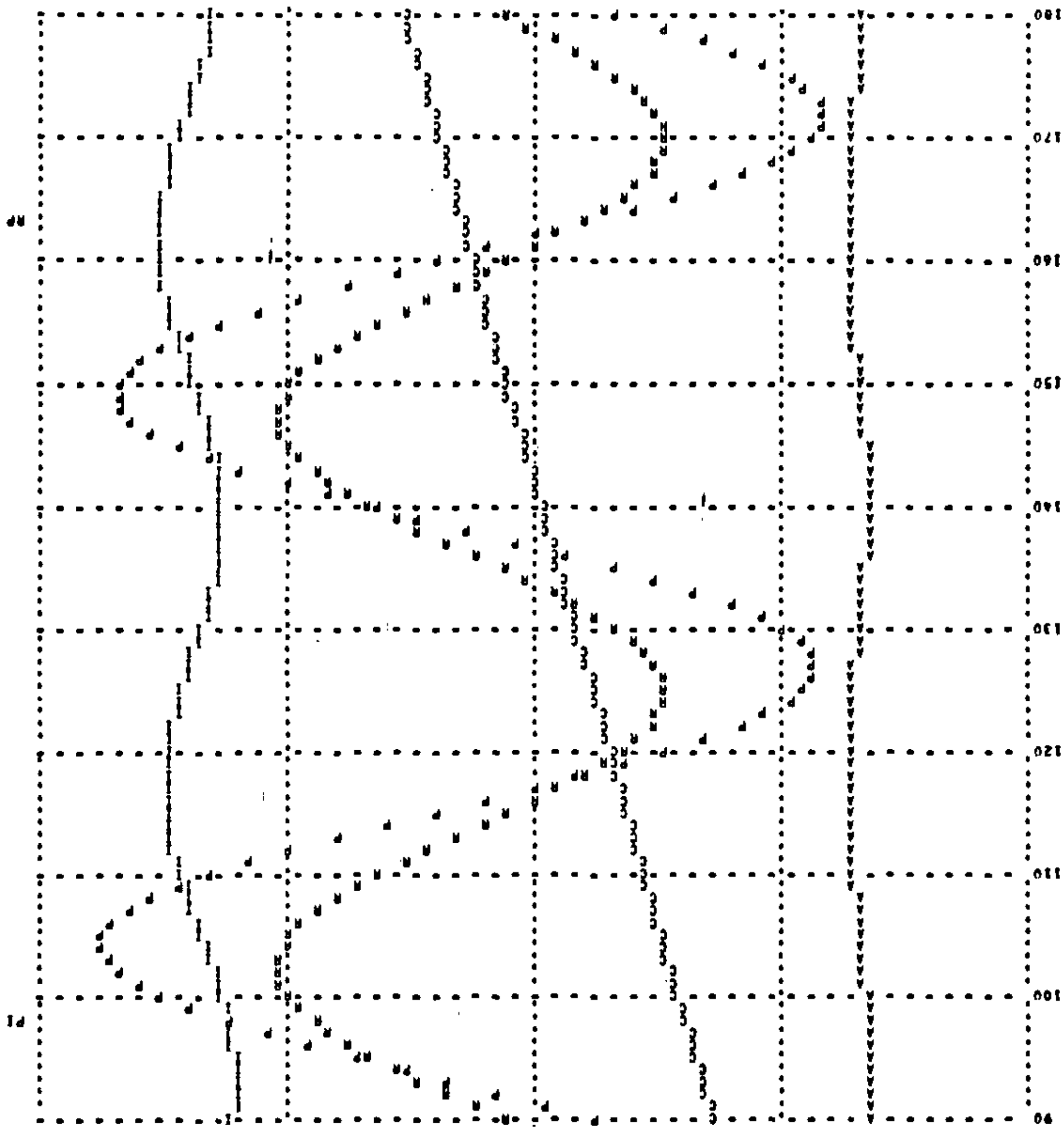
Reduced Delays, and Increased AIL





Basic Model with Increased AIL





90
100
110
120
130
140
150
160
170
180
P1
RP

BIBLIOGRAPHY

1. Alcalay, Jack A., and Buffa, Elwood S., "A Proposal for a General Model of a Production System," International Journal for Production Research, March 1963.

2. Ansoff, H. Igor, and Slevin, Dennis P., "An Appreciation of Industrial Dynamics," Management Science, Vol. 14A, No. 7, March 1968.

3. Ansoff, H. Igor, and Slevin, Dennis P., "Comments on Professor Forrester's 'Industrial Dynamics - After the First Decade'," Management Science, Vol. 14A, No. 9, 1968.

4. Arrow, Kenneth J., Harris, Theodore, and Marshak, Jacob, "Optimal Inventory Policy," Econometrica, Vol. 19, No. 3, July 1952.

5. Bather, J. A., "A Continuous Time Inventory Model," Journal of Applied Probability, No. 3, 1966.

6. Belsen, D. L., Fleischer, Gerald A., "Optimal Farm-to-Market Transportation and Storage of Certain Agriculture Commodities: Bogota Columbia," Engineering Economist, Vol. 15, No. 4, Summer 1970.

7. Bomberger, Earl E., "A Dynamic Programming Approach to a Lot Size Scheduling Problem," Management Science, Vol. 12, No. 11, July 1966.

8. Bulinskaya, E. V., "Optimum Inventory Policies with a Convex Ordering Cost Function," Theory of Probability and Its Application, Vol. 12, No. 1, 1967.

9. Bulinskaya, E. V., "Some Results Concerning Optimum Inventory Policies," Theory of Probability and Its Application, Vol. 9, No. 3, 1964.

10. Carlson, Bruce R., "An Industrialist Views Industrial Dynamics," Industrial Management Review, Fall 1964.

11. Collier, James D., and McCarthy, Thomas J., "Short Range Forecasting for Inventory Management with Probability Tree Analysis and Monte Carlo Simulation," Production and Inventory Management, 3rd Qtr., 1968.

12. Dooley, A. R., McGarrah, R. E., McKenney, J. L., Rosenbloom, R. S., Skinner, C. W., and Thruston, P. H., Casebook in Production Management-Basic Problems, Concepts, and Techniques, John Wiley and Sons, Inc., New York, 1964.

13. Eilon, S., "Obsolescense of Commodities Which are Subject to Deterioration in Store," Management Science, Vol. 9, No. 4, July 1963.

14. Eppen, Gary D., Gould, F. J., and Pashigian, B. Peter, "Extensions of the Planning Horizon Theorem in the Dynamic Lot Size Model," Management Science, Vol. 15, No. 5, January 1969.

15. Fey, Willard R., "An Industrial Management Case Study," Industrial Management Review, Fall 1962.

16. Forrester, Jay W., Industrial Dynamics, The M. I. T. Press, Cambridge, Mass., 1961.

17. Forrester, Jay W., "Industrial Dynamics - After the First Decade," Management Science, Vol. 14A, No. 7, March 1968.

18. Forrester, Jay W., "Industrial Dynamics - A Response to Ansoff and Slevin," Management Science, Vol. 14A, No. 9, May 1968.

19. Forrester, Jay W., "Market Growth as Influenced by Capital Investment," Industrial Management Review, Winter 1968.

20. Forrester, Jay W., Urban Dynamics, The M. I. T. Press, Cambridge, Mass., 1969.

21. Gleason, John M., "A Computational Variation of the Wagner-Whitin Algorithm: An Alternative to the E. O. Q.," Production and Inventory Management, 1st Qtr., 1971.

22. McGarrah, R. E., "Production Planning," Journal of Industrial Engineering, Vol. 7, No. 6, Nov-Dec 1956.

23. Noth, Mahendra, and Kamran, Kunal, "Inventory System with Built-in Controls," Technical Papers, Twenty-First Institute Conference and Convention, American Institute of Industrial Engineers, 1970.

24. Pugh, Alexander L., III, Dynamo User's Manual, Second Edition, The M. I. T. Press, Cambridge, Mass., March 1966.

25. Reed, Phillip A., and Hanken, A. F., "The Systems Approach to Inventory," Production and Inventory Management, Vol. 7, No. 4, October 1966.

26. Reisman, Arnold, and Buffa, Elwood S., "A General Model for Production and Operations Research, Management Science, Vol. 9, No. 1 September 1964.

27. Richardson, Stephen K., "Using Simulation and Remote Computing for Production and Inventory Control," Production and Inventory Management, 4th Qtr., 1968.

28. Riggs, James L., Productions Systems: Planning, Analysis, and Control, John Wiley and Sons, Inc., New York, 1970.

29. Roberts, Edward B., "New Directions in Industrial Dynamics," Industrial Management Review, Fall 1964.

30. Serma, Raul, "Dynamics of Unemployment in Developing Cities: An Industrial Dynamics Study," M. S. Thesis, Georgia Institute of Technology, December 1970.

31. Simon, Herbert Alexander, "Application of Servomechanism Theory To Production Control," Models of Man, John Wiley and Sons, Inc., 1957.

32. Swanson, Carl V., "Design of Resource Control and Marketing Policies Using Industrial Dynamics," Industrial Management Review, Spring 1969.

33. Thomas, Adin B., Inventory Control in Production and Manufacturing, Caveners Publishing Company, Inc., Boston, Massachusetts, 1969.

34. Tuite, Matthew F., and Anderson, William A., "A Comparison of Lot Size Algorithms Under Fluctuating Demand Conditions," Production and Inventory Management, 4th Qtr., 1968.

35. Vergin, Roger C., "Production Scheduling Under Seasonal Demand," Journal of Industrial Engineering, Vol. 17, No. 5, 1966.

36. Wagner, Harvey M., and Whitin, Thomson M., "Dynamic Version of the Economic Lot Size Model," Management Science, Vol. 5, No. 1 October 1958.

37. Zangwill, Willard I., "A Backlogging Model and a Multi-Echelon Model of a Dynamic Economic Lot Size Production System - A Network Approach," Management Science, Vol. 15, No. 9, May 1969.

38. Zangwill, Willard I., "A Deterministic Multiproduct, Multifacility Production and Inventory Model," Tech Roport No. 3, ONR Contract 225(77) and WMSI Grant, I. E. Dept, Stanford University, May 1965, 105 pp.

39. "Cabbage, Celery, Lettuce, and Tomatoes Laboratory Tests of Storage Methods," Market Research Report No. 402, United States Department of Agriculture, February 1968.

40. "Effect of Biphenyl Treatment and Carton Ventilation on Decay and Spoilage of California Lemons in Overseas Shipments," Marketing Research Report No. 500, United States Department of Agriculture October 1961.

41. "Extending the Storage Life of Cabbage, Celery, Lettuce, and Tomatoes Aboard a Navy Supply Ship," Marketing Research Report No. 336, United States Department of Agriculture, June 1959.

42. "Polyethylene Box Liners for Storage of Golden Delicious Apples," Marketing Research Report No. 461, United States Department of Agriculture, October 1966.

43. "Programmers Manual 'A Dynamo-To-Algol Translator'," Burroughs B550 Information Processing System, 8th Ed., Burroughs Corp., Management Systems Development Department, Detroit, Michigan, June 1971.

44. "Storage and Shelflife of Packaged Blueberries," Marketing Research Report No. 617, U. S. Dept. of Agriculture, July 1963.

45. "Supplement to Storage of Behavior of Lemons From the Desert Areas of Arizona and California," Supplement to Marketing Research Report No. 310, United States Dept. of Agriculture, Nov. 1961.